Name: ____________________

Instructor: ____________________

Time of your class: __________

UMKC Department of Mathematics and Statistics

Math 210 CALCULUS I
Common Final Examination

Saturday, May 5, 2018

INSTRUCTIONS:

1. Write your name, the name of your instructor and the time your class meets on top.
2. Only scientific calculators with no integration capabilities may be used for this examination. Graphing calculators and electronic communication devices are NOT permitted in the room.
3. Make sure that your test contains all problems indicated below on this cover page.
4. On page 1 ONLY: Multiple choice problems; full credit will be given for a correct answer and no credit for an incorrect answer; no work needs to be shown.
5. On all other pages: **All work MUST be shown to receive full credit.**
6. Unless specifically asked not to simplify, all answers need to be simplified completely.
7. EXACT answers are expected, NO approximations (unless otherwise specified).
8. Write your final answer in the box provided.

Problem	Possible	Earned
1	16	
2	4	
3	8	
4	11	
5	10	
6	16	
7	8	
8	9	
9	8	
10	9	
11	14	
12	10	
13	10	
14	10	
15	10	
16	9	
17	12	
18	6	
19	10	
20	10	
Total	200	

1. (16 pts) Determine the following limits. Choose "DNE" (Does Not Exist) as your answer only if your function has NO limit, finite or infinite. **Circle one of the indicated choices**. No work needs to be shown.

(a) (4 pts) $\lim_{x\to 1} \dfrac{\sqrt{x}-1}{x-1} =$

A. $\dfrac{1}{2}$

B. 1

C. 0

D. 2

E. ∞

F. DNE

(b) (4 pts) $\lim_{x\to 4^+} \dfrac{x-7}{(x-4)^2} =$

A. -1

B. 1

C. 0

D. $-\infty$

E. ∞

F. DNE

(c) (4 pts) $\lim_{x\to \infty} \dfrac{3x\sqrt{x}+4}{x^2+x+1}$

A. 4

B. 3

C. 0

D. ∞

E. $-\infty$

F. DNE

(d) (4 pts) $\lim_{x\to 2} \left(\dfrac{x^2-3+\cos^2(x-2)}{1-x+\sin^3(x-2)}\right)^{x+1}$

A. 1

B. 27

C. -1

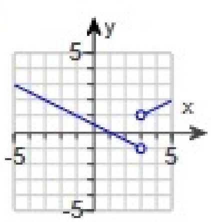
D. -8

E. ∞

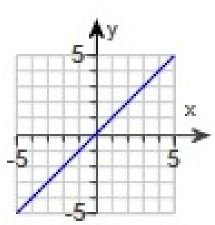
F. DNE

2. (4 pts) Decide which of the following graphs describes a function $f(x)$ such that $f(x)$ is **not** continuous at $x=3$ and yet $\lim_{x\to 3} f(x)$ exists. Mark one of the choices.

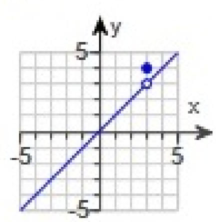
○ A.



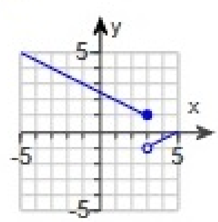
○ B.



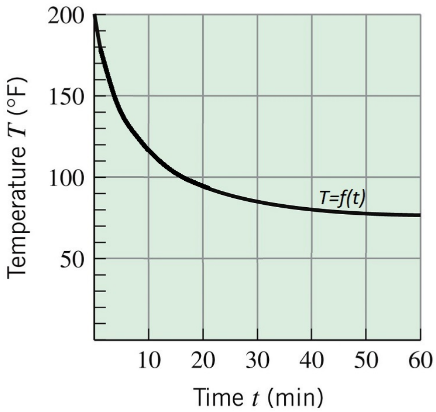
○ C.



○ D.



3. (8 pts) An object has been heated and now it is cooling down. The graph below represents its temperature T as a function of time t. Use this graph to estimate the **instantaneous rate of change** of T with respect to t at the times when $t = 10$ min and $t = 56$ min. Show your work.



the instantaneous rate of change at $t = 10$ is approximately: ____________

the instantaneous rate of change at $t = 56$ is approximately: ____________

4. (11 pts) Consider the function $f(x) = \frac{x^2}{4} + 4\sqrt{x} + \frac{8}{\sqrt{x}}$. Find the slope of the line tangent to the graph of $f(x)$ at the point with $x = 4$.

slope=

5. (10 pts) Compute $f'(2)$ and $f''(2)$ for the function $f(x) = (\ln x)^4$.

Make sure to evaluate the derivatives at $x = 2$ and to simplify your answer, but do not approximate.

$f'(2) =$
$f''(2) =$

6. (16 pts) Compute the derivatives of the following functions. Do NOT simplify.

(a) (6 pts) $g(x) = x^3 \cos(x^2)$

(b) (10 pts) $h(x) = \left(\dfrac{e^{2x}}{\tan(x)} \right)^{100}$

7. (8 pts) Use the limit DEFINITION of the derivative to show that the derivative of the function $f(x) = 3x^2 + x$ is $f'(x) = 6x + 1$.

8. (9 pts) Consider the implicit equation $x^3 + 5y^2x = y^3$. Find $\dfrac{dy}{dx}$.

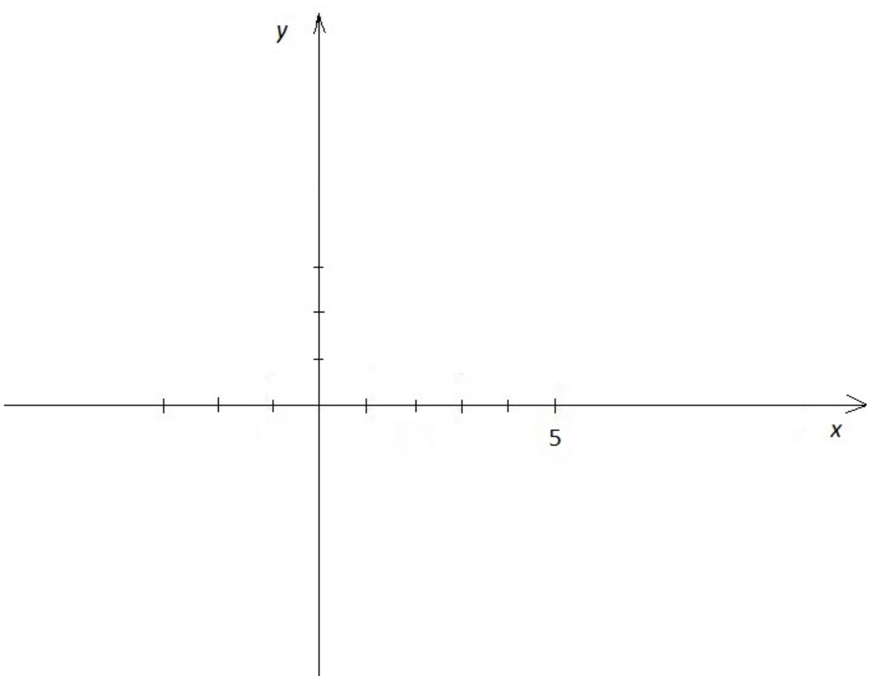
$\dfrac{dy}{dx} =$

9. (8 pts) Use L'Hôpital's Rule to compute the limit below. Show all work.

(a) $\displaystyle\lim_{x \to -4} \frac{\sin(\pi x)}{x^2 - 16} =$

10. (9 pts) Sketch below the graph of a function that is differentiable on $(-\infty, \infty)$ and satisfies all of the following properties:

$$f'(0) < 0, \qquad f''(0) < 0, \qquad f'(5) > 0, \qquad f''(5) > 0$$



11. (14 pts) Determine the x-coordinates of the points of **absolute maximum** and **absolute minimum** of the function below over the given intervals. If such points do not exist, write DNE. Make sure to show enough work in order justify all answers.

(a) (9 pts) $f(x) = x^2 - 8\ln x$, over the interval $[1, 10]$.

point(s) of absolute minimum at $x =$
point(s) of absolute maximum at $x =$

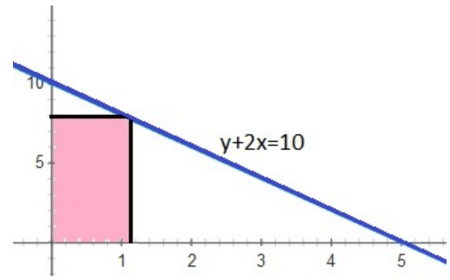
(b) (5 pts) $f(x) = x^2 - 8\ln x$, over the interval $(0, \infty)$.

point(s) of absolute minimum at $x =$
point(s) of absolute maximum at $x =$

12. (10 pts) Determine the intervals on which the function $f(x) = x^4 - x^3 + 4$ is concave up or concave down. Identify any inflection points.

concave down on:
concave up on:
inflection point(s) at $x =$

13. (10 pts) A rectangle is constructed with one vertex at the origin, one side on the positive x-axis, one side on the positive y-axis and the vertex opposite to the origin on the line $y+2x=10$. Denote by x and y the lengths of the two sides. What values of x and y maximize the area of the rectangle? What is the maximum area?

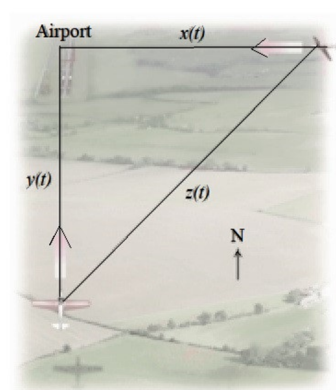


$x =$

$y =$

Max area=

14. (10 pts) Two small planes approach an airport, one flying due west at 125 mi/hr and the other one flying due north at 150 mi/hr. Assuming that they fly at the same constant elevation, how fast is the distance between the planes changing when the westbound plane is 50 mi from the airport and the northbound plane is 120 mi from the airport?



15. (10 pts) Compute the integral: $I = \int \left(\frac{2}{x^{20}} + \frac{x^{20}}{3} - \sec^2 x \right) dx.$

$I =$

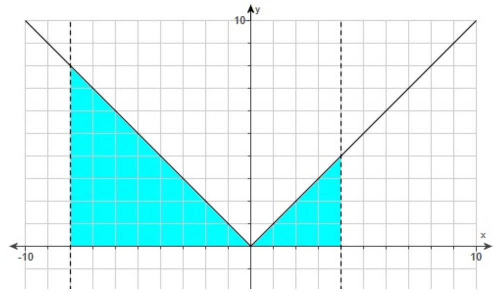
16. (9 pts) Use a change of variable (u-substitution) to compute the integral $I = \int_0^5 \frac{x^2}{x^3+5}\, dx.$

$I =$

17. (12 pts) An object is moving along a line with acceleration $a(t) = 4\sin(3t)$. Find the position function $s(t)$, given the initial velocity $v(0) = 1$ and the initial position $s(0) = 11$.

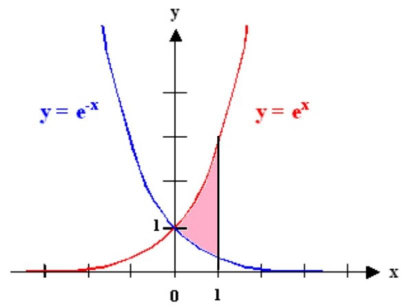
$s(t) =$

18. (6 pts) Compute the integral $I = \int_{-8}^{4} |x|\,dx$. You may use geometry. (See the picture below.)



$I =$

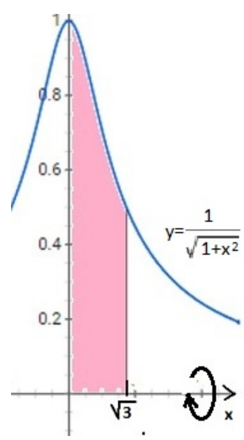
19. (10 pts) Find the area between $y = e^x$, $y = e^{-x}$, $x = 0$ and $x = 1$.



AREA=

20. (10 pts) Find the volume of the solid generated by revolving around the x-axis the following region:

$$\text{the region under } y = \frac{1}{\sqrt{1+x^2}} \text{ over the interval } [0, \sqrt{3}].$$



VOLUME=