Name: ____________________________

Instructor: ____________________________

Time of your class: __________

UMKC Department of Mathematics and Statistics

Math 210 CALCULUS I
Common Final Examination

Saturday, December 9, 2017

INSTRUCTIONS:

1. Write your name, the name of your instructor and the time your class meets on top.
2. Only scientific calculators with no integration capabilities may be used for this examination. Graphing calculators and electronic communication devices are NOT permitted in the room.
3. Make sure that your test contains all problems indicated below on this cover page.
4. On page 1 ONLY: Multiple choice problems; full credit will be given for a correct answer and no credit for an incorrect answer; no work needs to be shown.
5. On all other pages: **All work MUST be shown to receive full credit.**
6. Unless specifically asked not to simplify, all answers need to be simplified completely.
7. EXACT answers are expected, NO approximations (unless otherwise specified).
8. Write your final answer in the box provided.

Problem	Possible	Earned
1	12	
2	8	
3	10	
4	8	
5	10	
6	16	
7	8	
8	8	
9	10	
10	12	
11	11	
12	11	
13	10	
14	10	
15	10	
16	9	
17	12	
18	4	
19	12	
20	9	
Total	200	

1. (12 pts) Determine the following limits. Choose "DNE" (Does Not Exist) as your answer only if your function has NO limit, finite or infinite. **Circle one of the indicated choices**. No work needs to be shown.

(a) (4 pts) $\lim_{x \to -\infty} \frac{x^2+3x}{x+1} =$

A. 3

B. 1

C. -1

D. ∞

E. $-\infty$

F. DNE

(b) (4 pts) $\lim_{x \to \infty} \frac{3x+1}{1+17x+x^2} =$

A. 3

B. 1

C. $\frac{3}{17}$

D. $\frac{1}{17}$

E. 0

F. ∞

(c) (4 pts) $\lim_{x \to 0^-} \frac{x^2+3x}{x^4} =$

A. 3

B. 1

C. 0

D. ∞

E. $-\infty$

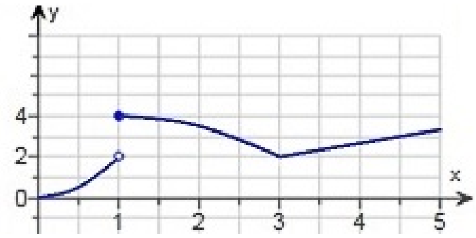
F. DNE

2. (8 pts) Consider the function: $f(x) = \begin{cases} \frac{x^2-7x+10}{x-2} & \text{if } x > 2 \\ x^2 & \text{if } x < 2 \end{cases}$.

Compute the following limits. Write "DNE" if a limit does not exist.

$\lim_{x \to 2^+} f(x) =$
$\lim_{x \to 2^-} f(x) =$
$\lim_{x \to 2} f(x) =$

3. (10 pts) Consider the function $f(x)$ whose graph is shown:



(a) (3 pts) Find all the values of x in the interval $(0,5)$ at which the function $f(x)$ is not continuous.

$x =$

(b) (3 pts) Find all the values of x in the interval $(0,5)$ at which the function $f(x)$ is not differentiable.

$x =$

(c) (4 pts) Indicate which of the following statements is true and which is false. Write either "True" or "False" next to each statement, as appropriate.

i. $f'(2) > 0$;

ii. $f''(2) > 0$;

iii. $f'(4) = 0$;

iv. $f''(4) = 0$.

4. (8 pts) Find the instantaneous rate of change of the function $f(x) = 2x^3 + 12\sqrt{x}$ at $x = 4$.

$\text{rate}\Big|_{x=4} =$

5. (10 pts) Compute $f'(1)$ and $f''(1)$ for the function $f(x) = e^{x^3}$.

Make sure to evaluate the derivatives at $x = 1$ and to simplify your answer.

$f'(1) =$

$f''(1) =$

6. (16 pts) Compute the derivatives of the following functions. Do NOT simplify.

(a) (9 pts) $g(x) = \Big(\tan(x)\ln(x)\Big)^{100}$

(b) (7 pts) $h(x) = \dfrac{\sin(3x)}{2x+7}$

7. (8 pts) Use the limit DEFINITION of the derivative to show that the derivative of the function $f(x) = \dfrac{1}{x}$ is $f'(x) = -\dfrac{1}{x^2}$.

8. (8 pts) Consider the implicit equation $x^3 + y^3 = 30xy$. Find $\dfrac{dy}{dx}$.

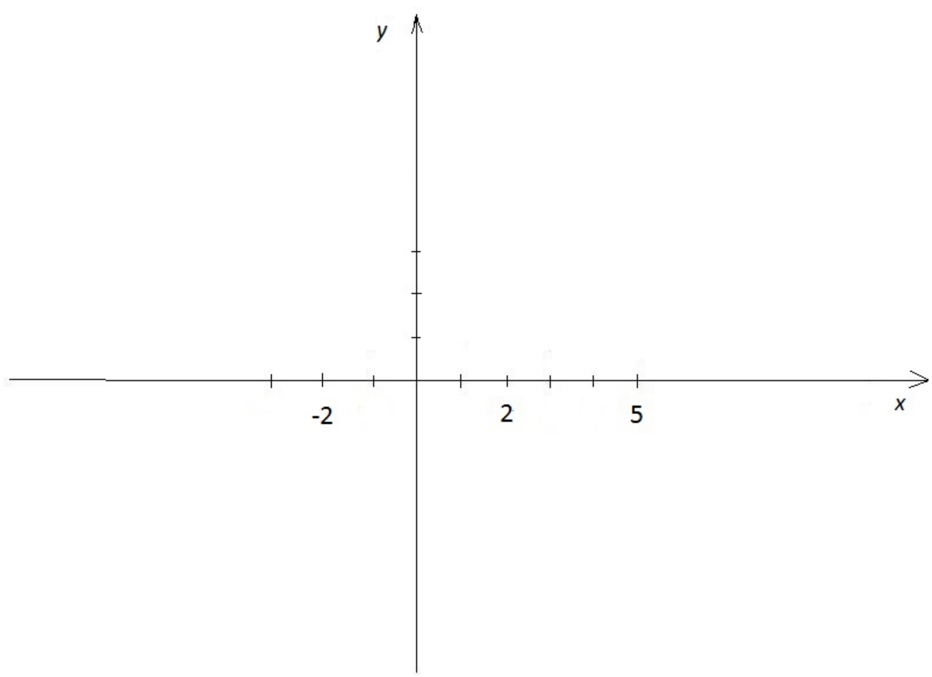
$\dfrac{dy}{dx} =$

9. (10 pts) Use L'Hôpital's Rule to compute the limit below. Show all work.

$$\lim_{x \to 0} \frac{e^x - \sin x - 1}{x^4 + 5x^3 + 12x^2} =$$

10. (12 pts) Sketch the graph of a function that satisfies all of the following properties. Draw any asymptotes using dashed lines.

(a) $f(x)$ is continuous and differentiable everywhere, except at $x = 2$;
(b) $\lim_{x \to 2^-} f(x) = \infty$ and $\lim_{x \to 2^+} f(x) = \infty$;
(c) $\lim_{x \to -\infty} f(x) = 3$ and $\lim_{x \to \infty} f(x) = \infty$.
(d) $f'(-2) = 0$ and $f'(5) = 0$.
(e) $f'(x) < 0$ on the intervals $(-\infty, -2)$ and $(2, 5)$, and $f'(x) > 0$ on the intervals $(-2, 2)$ and $(5, \infty)$.



11. (11 pts) Find the interval(s) on which f is increasing or decreasing and the x-coordinates of the point(s) of local maximum/minimum for the function $f(x) = \frac{x^4}{4} - x^3 + x^2$.

$f(x)$ is increasing on:
$f(x)$ is decreasing on:
point(s) of local (relative) minimum at $x =$
point(s) of local (relative) maximum at $x =$

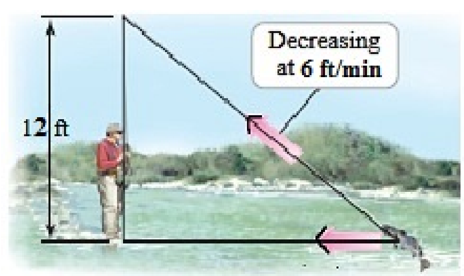
12. (11 pts) Determine the intervals on which the function $f(x) = \ln(5x^2 + 1)$ is concave up or concave down. Identify any inflection points.

concave down on:
concave up on:
inflection point(s) at $x =$

13. (10 pts) Find positive numbers x and y satisfying the equation $xy = 28$ such that the sum $4x + y$ is as small as possible.

$x =$
$y =$

14. (10 pts) An angler hooks a trout and reels in his line at 6 ft/min. Assume the tip of the fishing rod is 12 ft above the water and directly above the angler, and the fish is pulled horizontally directly toward the angler (see figure). Find the horizontal speed of the fish when it is 5 ft from the angler.



horizontal speed=

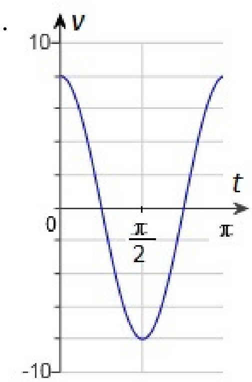
15. (10 pts) Compute the integral: $I = \int \left(\frac{4}{1+x^2} + \frac{2}{\sqrt{x}} + \frac{x^3}{10} \right) dx.$

$I =$

16. (9 pts) Use a change of variable (u-substitution) to compute the integral $I = \int x^2 e^{x^3+7}\, dx.$

$I =$

17. (12 pts) The velocity in m/sec of a particle moving along the x-axis is $v(t) = 8\cos(2t)$.



(a) (6 pts) Find the displacement over the interval $0 \leq t \leq \frac{\pi}{2}$.

DISPLACEMENT=

(b) (6 pts) Find the distance travelled between $t = 0$ and $t = \frac{\pi}{2}$.

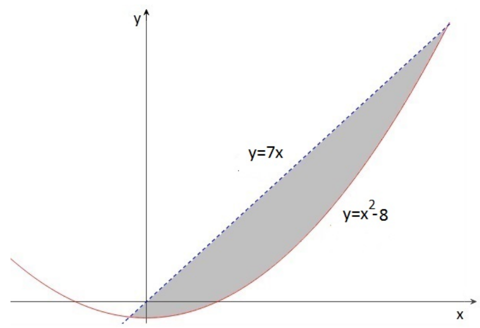
DISTANCE=

18. (4 pts) Consider the following limit of Riemann sum of a function f over the interval $[1,2]$. Identify f and express the limit as a definite integral:

$$L = \lim_{\Delta \to 0} \sum_{k=1}^{n} x_k^* \ln(x_k^*) \Delta x_k$$

$L =$

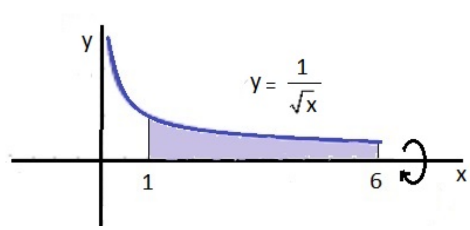
19. (12 pts) In the picture below, the equation of the parabola is $y = x^2 - 8$ and the equation of the dashed line is $y = 7x$. Determine the area of the shaded region.



AREA=

20. (9 pts) Find the volume of the solid generated by revolving the region bounded by the following lines and curves about the x-axis:

$$y = \frac{1}{\sqrt{x}},\ y = 0,\ x = 1,\ x = 6$$



VOLUME=