Name: Key

Instructor: ______

Time of your class: ______

UMKC Department of Mathematics and Statistics

Math 210 CALCULUS I

Common Final Examination

Saturday, May 9, 2015

INSTRUCTIONS:

1. Write your name, the name of your instructor and the time your class meets on top.
2. Only scientific calculators with no integration capabilities may be used for this examination. Graphing calculators and electronic communication devices are NOT permitted in the room.
3. Make sure that your test contains all problems indicated below on this cover page.
4. On page 1 ONLY: Multiple choice problems; full credit will be given for a correct answer and no credit for an incorrect answer; no work needs to be shown.
5. On all other pages: **All work MUST be shown to receive full credit.**
6. Unless specifically asked not to simplify, all answers need to be simplified completely.
7. EXACT answers are expected, NO approximations (unless otherwise specified).
8. Write your final answer in the box provided.

Problem	Possible	Earned
1	20	
2	5	
3	9	
4	8	
5	20	
6	6	
7	9	
8	9	
9	5	
10	8	
11	7	
12	9	
13	9	
14	8	
15	10	
16	9	
17	10	
18	6	
19	10	
20	5	
21	9	
22	9	
Total	200	

1. (20 pts) Determine the following limits. Choose "DNE" (Does Not Exist) as your answer only if your function has NO limit, finite or infinite. **Circle one of the indicated choices.** No work needs to be shown.

(a) (5 pts) $\lim_{x\to\infty} \frac{12-7x^5}{2x^3-10x+2} =$

A. 6

B. $-\frac{7}{2}$

C. DNE

D. 0

E. ∞

F. $-\infty$

(b) (5 pts) $\lim_{x\to\infty} \frac{\sqrt{x^2+3}}{3x+1} =$

A. $\frac{\sqrt{3}}{3}$

B. $\sqrt{3}$

C. $\frac{1}{3}$

D. 0

E. ∞

F. 1

(c) (5 pts) $\lim_{x\to\infty} \ln\left(\frac{2}{x^2}\right) =$

A. 2

B. 1

C. DNE

D. 0

E. ∞

F. $-\infty$

(d) (5 pts) $\lim_{x\to 4^+} \frac{x+4}{x^2-16} = \lim_{x\to 4^+} \frac{1}{x-4}$

$\frac{1}{0^+}$

A. 8

B. -16

C. DNE

D. 0

E. ∞

F. $-\infty$

2. (5 pts) Find a value of the constant k, if possible, that will make the function f continuous everywhere.

$$f(x) = \begin{cases} kx^2 & \text{for } x \le 3 \\ 4x + k & \text{for } x > 3 \end{cases}$$

$$9k = 12 + k$$

$$8k = 12 \rightsquigarrow k = \frac{12}{8} = \frac{3}{2}$$

$\boxed{k = \frac{3}{2}}$

3. (9 pts) Find the equation of the line tangent to the graph of $y = \sqrt{x} - 3$ at $x = 1$.

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \qquad X = 1 \rightsquigarrow y = 1 - 3 = -2$$

$$m = \left.\frac{dy}{dx}\right|_{X=1} = \frac{1}{2} \qquad (1, -2)$$

$$y + 2 = \frac{1}{2}(X - 1) = \frac{1}{2}X - \frac{1}{2}$$

$$y = \frac{1}{2}X - \frac{1}{2} - 2 = \frac{1}{2}X - \frac{5}{2}$$

$\boxed{y = \frac{1}{2}X - \frac{5}{2}}$

4. (8 pts) Use the limit DEFINITION of the derivative to show that the derivative of the function $f(x) = \frac{2}{x}$ is $f'(x) = -\frac{2}{x^2}$.

$$f'(X) = \lim_{h \to 0} \frac{f(X+h) - f(X)}{h} = \lim_{h \to 0} \frac{\frac{2}{X+h} - \frac{2}{X}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2X - 2(X+h)}{(X+h)X}}{h} = \lim_{h \to 0} \frac{\frac{2X - 2X - 2h}{(X+h)X}}{h}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{(X+h)X \cdot \cancel{h}} = \lim_{h \to 0} \frac{-2}{(X+\underset{\downarrow 0}{h})X} = -\frac{2}{X^2}$$

5. Compute the derivatives of the following functions; do not simplify the answer.

(a) (8 pts) $f(x) = \ln(3x^2 + 7) + \sin^2(x)$

$$\boxed{f'(x) = \frac{1}{3x^2+7} \cdot 6x + 2\sin(x)\cos(x)}$$

(b) (6 pts) $g(x) = e^{5x} \sin(3x)$

$$\boxed{g'(x) = 5e^{5x} \cdot \sin(3x) + e^{5x} \cdot 3 \cdot \cos(3x)}$$

(c) (6 pts) $h(x) = \dfrac{5x^4 + 3}{\tan x}$

$$\boxed{h'(x) = \frac{20x^3 \cdot \tan x - (5x^4+3)\sec^2 x}{\tan^2 x}}$$

6. (6 pts) Let $f(x) = x^8 + \dfrac{1}{x^8}$. Compute $f'(1)$ and $f''(1)$.

$$f(x) = x^8 + x^{-8}$$
$$f'(x) = 8x^7 + (-8)x^{-9}$$
$$f''(x) = 8 \cdot 7x^6 + (-8)(-9)x^{-10}$$

$$f'(1) = 8 + (-8) = 0$$
$$f''(1) = 8 \cdot 7 + 8 \cdot 9 = 8 \cdot 16 = 128$$

$\boxed{f'(1) = 0}$

$\boxed{f''(1) = 128}$

7. (9 pts) Use implicit differentiation to find the derivative $\frac{dy}{dx}$ at the point with $x = \frac{\pi}{2}$ and $y = 1$, given that

$$\cos(xy^6) = 2x - \pi$$

$$\frac{d}{dX}\cos(Xy^6) = \frac{d}{dX}(2X - \pi)$$

$$-\sin(Xy^6)\cdot\left(1\cdot y^6 + X\cdot 6y^5\cdot\frac{dy}{dX}\right) = 2$$

$X = \frac{\pi}{2}, y = 1$

$$-\sin\left(\frac{\pi}{2}\right)\cdot\left(1 + \frac{\pi}{2}\cdot 6\cdot 1\cdot\frac{dy}{dX}\right) = 2$$

$$-1\cdot\left(1 + 3\pi\frac{dy}{dX}\right) = 2 \qquad \frac{dy}{dX} = -\frac{2-1}{3\pi} = \frac{-1}{\pi}$$

$$\boxed{\left.\frac{dy}{dx}\right|_{(x,y)=(\frac{\pi}{2},1)} = -\frac{1}{\pi}}$$

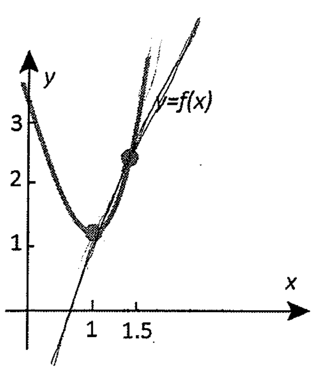
8. (9 pts) Find the limit and provide a complete justification:

$$\lim_{x\to 0}\frac{\cos(3x) - x - 1}{e^{7x} - 1} = \lim_{X\to 0}\frac{-3\cdot\sin(3X) - 1}{e^{7X}\cdot 7} = \frac{-1}{7}$$

L'H $\frac{0}{0}$

$$\boxed{-\frac{1}{7}}$$

9. (5 pts) The picture below shows the graph of a function $f(x)$. If $f(1) = 1.13$ and $f(1.5) = 2.45$, use this information to **approximate** the value of $f'(1.5)$. Indicate your reasoning.



$$f'(1.5) \approx \frac{f(1) - f(1.5)}{1 - 1.5} = \frac{f(1.5) - f(1)}{1.5 - 1}$$

$$= \frac{2.45 - 1.13}{.5}$$

$$= \frac{1.32}{\frac{1}{2}} = 1.32\cdot 2 = 2.64$$

$$\boxed{f'(1.5) \simeq 2.64}$$

(The symbol $\simeq$ means "approximately equal to")

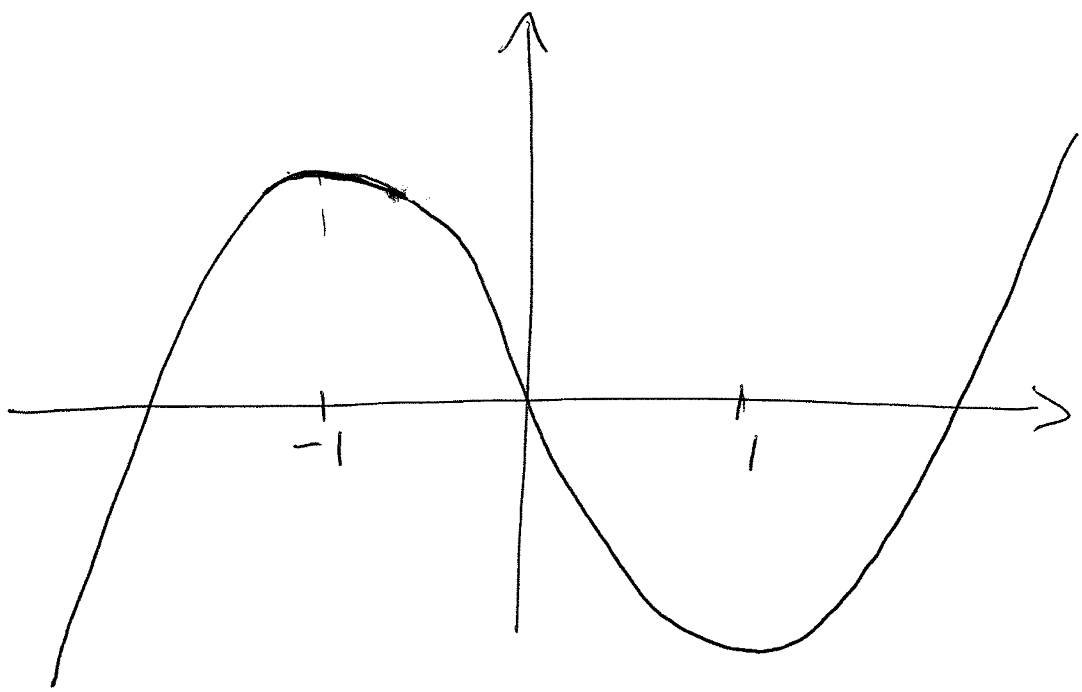
10. (8 pts) Sketch below the graph of a function $f(x)$ that satisfies ALL of the following properties:

(a) $f(x)$ is continuous and differentiable everywhere;

(b) $f'(x) = 0$ for $x = 1$ and $x = -1$;

(c) $f'(x) < 0$ for $-1 < x < 1$;

(d) $f'(x) > 0$ for $x < -1$ and for $x > 1$.



11. (7 pts) The function $s(t)$ below describes the position of a particle moving along a coordinate line, where s is in feet and t in seconds:

$$s(t) = 3t^3 - 9t, \;\; t \geq 0$$

(a) (4 pts) Find the velocity and the acceleration functions.

$v(t) =$ $9t^2 - 9$

$a(t) =$ $18t$

(b) (3 pts) At what times t is the particle stopped?

$9t^2 - 9 = 0$

$t^2 = 1 \qquad t = \pm 1 \rightarrow t = 1$

$(t \geq 0)$

$t =$ 1

12. (9 pts) Find the absolute maximum and minimum values of f on the given closed interval, and state where those values occur.

$$f(x) = x^3 + 3x^2 - 9x, \qquad [-2, 2]$$

$f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x+3)(x-1)$

$f'(x) = 0 \rightsquigarrow \cancel{x = -3}, x = 1$

↳ not in $[-2, 2]$.

$f(1) = 1 + 3 - 9 = -5$ min

$f(2) = 8 + 12 - 18 = 2$

$f(-2) = -8 + 12 + 18 = 22$ max.

$\boxed{\text{maximum value} = 22 \text{ at } x = -2}$

$\boxed{\text{minimum value} = -5 \text{ at } x = 1}$

13. (9 pts) Consider the function $f(x) = x^3 + 5x^2$. Determine all intervals where f is strictly increasing.

$f'(x) = 3x^2 + 10x = x(3x + 10)$

$f'(x) = 0 \rightsquigarrow x = 0, x = -\frac{10}{3}$.

$+ \quad -\frac{10}{3} \quad - \quad 0 \quad +$

$\nearrow \quad \searrow \quad \nearrow$

$\boxed{\left(-\infty, -\frac{10}{3}\right) \text{ and } (0, \infty)}$

14. (8 pts) The second derivative of a function $f(x)$ is $f''(x) = (x-1)^2(x+2)(x-3)$. Determine all intervals where f is concave up.

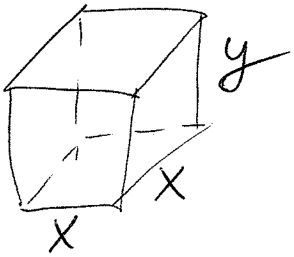
$f''(x) = 0 \rightsquigarrow x = 1, x = -2, x = 3$.

$+ \quad -2 \quad - \quad 1 \quad - \quad 3 \quad +$

$\cup \quad \frown \quad \frown \quad \cup$

$\boxed{(-\infty, -2) \text{ and } (3, \infty)}$

15. (10 pts) A closed rectangular container with a square base is to have a volume of 1152 in^3. The material for the top and bottom of the container will cost \$2 per in^2, and the material for the sides will cost \$3 per in^2. Find the dimensions of the container of least cost.



$V = x^2 y \rightsquigarrow x^2 y = 1152$

$y = \frac{1152}{x^2}$

$C = 2x^2 \cdot 2 + 4xy \cdot 3 = 4x^2 + 12xy$

$C(x) = 4x^2 + 12x \cdot \frac{1152}{x^2} = 4x^2 + \frac{12 \cdot 1152}{x}$

$C'(x) = 8x - \frac{12 \cdot 1152}{x^2}$

$-$ | $+$, 12, min

$C'(x) = 0 \rightsquigarrow 8x = \frac{12 \cdot 1152}{x^2}$

$8x^3 = 12 \cdot 1152$

$x^3 = 1728 \rightsquigarrow x = 12 \rightsquigarrow y = \frac{1152}{12^2} = 8$

length of edge of base = 12 in

height = 8 in

16. (9 pts) Solve the following related rates problem: Find $\frac{dx}{dt}$ given that

$$4x^2 + 9y^2 = 13, \qquad x = 1,\ y = -1, \text{ and } \frac{dy}{dt} = 10$$

$8x\frac{dx}{dt} + 18y\frac{dy}{dt} = 0$

$8 \cdot 1 \cdot \frac{dx}{dt} + 18 \cdot (-1) \cdot \frac{dy}{dt} = 0$

$\to 10$

$8\frac{dx}{dt} = 180$

$\frac{dx}{dt} = \frac{180}{8} = 22.5$

or $\frac{45}{2}$

$\frac{dx}{dt} = 22.5$

17. (10 pts) Evaluate the indefinite integral $\int \sqrt{2} - \frac{5}{x^8} + \sqrt[3]{x}\,dx$. Simplify the answer.

$$\sqrt{2}-5x^{-8}+x^{1/3}$$

$$\sqrt{2}x - 5\cdot\frac{x^{-7}}{-7} + \frac{x^{4/3}}{4/3} + C$$

$$5/7\, x^{-7} \text{ OK}$$

$$x^{4/3} \text{ OK.}$$

$$= \sqrt{2}x + \frac{5}{7}x^{-7} + \frac{3}{4}x^{4/3} + C$$

$$\boxed{\sqrt{2}x + \frac{5}{7x^7} + \frac{3}{4}\sqrt[3]{x^4} + C}$$

18. (6 pts) A particle moves with velocity $v(t) = \sin(4t)$ along an x-axis. Let $s(t)$ denote the position function. If $s(0) = 1$, find $s(t)$.

$$v(t) = s'(t) = \sin(4t).$$

$$\int \sin(4t)\,dt = -\frac{\cos(4t)}{4} + C$$

$$s(t) = -\frac{\cos(4t)}{4} + C$$

$$s(0) = 1 \rightsquigarrow -\frac{\cos(0)}{4} + C = 1$$

$$C = 1 + \frac{1}{4} = \frac{5}{4}$$

$$\boxed{s(t) = -\frac{\cos(4t)}{4} + \frac{5}{4}}$$

19. (10 pts) Evaluate the definite integral $I = \int_0^1 \frac{5x}{\sqrt{x^2+4}}\,dx$ using a u-substitution; show all work.

$$u = x^2+4$$
$$du = 2x\,dx$$
$$x\,dx = \frac{1}{2}du.$$

$$\int \frac{5x}{\sqrt{x^2+4}}\,dx = \int \frac{5\cdot\frac{1}{2}du.}{\sqrt{u}} =$$

$$= \frac{5}{2}\int u^{-1/2}du = \frac{5}{2.}\,\frac{u^{1/2}}{1/2} = \frac{5}{\cancel{2}}\cancel{2}\sqrt{u} + C$$

$$= 5\sqrt{x^2+4} + C$$

$$\int_0^1 \frac{5x}{\sqrt{x^2+4}}\,dx = 5\sqrt{x^2+4}\Big]_0^1 = 5\sqrt{5} - \underbrace{5\sqrt{4}}_{10}$$

$$\boxed{I = 5\sqrt{5} - 10}$$

20. (5 pts) Evaluate the indefinite integral $I = \int \frac{(\ln x)^2}{x}\, dx$.

$u = \ln x$

$du = \frac{1}{x} dx$

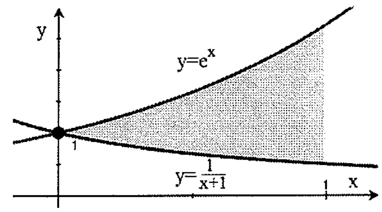
$\int u^2 du = \frac{u^3}{3} + C$

$= \frac{(\ln x)^3}{3} + C$

No work needs to be shown for this (easily guessed)

$\boxed{I = \frac{(\ln x)^3}{3} + C}$

21. (9 pts) Find the **area** between $y = e^x$ and $y = \frac{1}{1+x}$ on the interval $[0, 1]$. Do NOT approximate your answer.



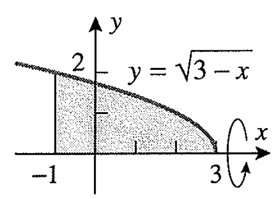
$\int_0^1 e^x - \frac{1}{1+x} dx =$

$= e^x - \ln|1+x| \Big/_0^1 = e^1 - \ln 2 - (e^0 - \underbrace{\ln 1}_{0})$

$= e - \ln 2 - 1$

$\boxed{\text{AREA} = e - \ln 2 - 1}$

22. (9 pts) Consider the region between curve $y = \sqrt{3-x}$ and the x-axis, over the interval $[-1, 3]$. Find the **volume** of the solid obtained when this region is rotated about the x-axis.



$V = \int_{-1}^{3} \pi (\sqrt{3-x})^2 dx = \int_{-1}^{3} \pi (3-x) dx$

$= \pi \cdot \left(3x - \frac{x^2}{2}\right) \Big/_{-1}^{3} = \pi\left(3 \cdot 3 - \frac{3^2}{2} - 3(-1) + \frac{(-1)^2}{2}\right)$

$= \pi\left(9 - \frac{9}{2} + 3 + \frac{1}{2}\right) = \pi\left(12 - \frac{8}{2}\right)$

$= \pi \cdot 8$

$\boxed{\text{VOLUME} = 8\pi}$