Name: KEY

Instructor:

Time of your class:

UMKC Department of Mathematics and Statistics

Math 210 CALCULUS I

Common Final Examination

Saturday, December 12, 2015

INSTRUCTIONS:

1. Write your name, the name of your instructor and the time your class meets on top.
2. Only scientific calculators with no integration capabilities may be used for this examination. Graphing calculators and electronic communication devices are NOT permitted in the room.
3. Make sure that your test contains all problems indicated below on this cover page.
4. On page 1 ONLY: Multiple choice problems; full credit will be given for a correct answer and no credit for an incorrect answer; no work needs to be shown.
5. On all other pages: **All work MUST be shown to receive full credit.**
6. Unless specifically asked not to simplify, all answers need to be simplified completely.
7. EXACT answers are expected, NO approximations (unless otherwise specified).
8. Write your final answer in the box provided.

Problem	Possible	Earned
1	20	
2	6	
3	6	
4	8	
5	18	
6	10	
7	10	
8	9	
9	5	
10	7	
11	9	
12	9	
13	10	
14	10	
15	10	
16	8	
17	10	
18	10	
19	9	
20	10	
21	6	
Total	200	

1. (20 pts) Determine the following limits. Choose "DNE" (Does Not Exist) as your answer only if your function has NO limit, finite or infinite. **Circle one of the indicated choices.** No work needs to be shown.

(a) (5 pts) $\lim\limits_{x\to\infty} \dfrac{x^2+2x}{3x^2+4} = \dfrac{1}{3}$

A. DNE
B. 1
C. $\frac{3}{7}$
D. $\frac{1}{3}$
E. $\frac{1}{4}$
F. ∞

(b) (5 pts) $\lim\limits_{x\to-2} \dfrac{3x}{(x+2)^2} = -\infty$

A. 3
B. $-\frac{3}{2}$
C. DNE
D. -6
E. ∞
F. $-\infty$

(c) (5 pts) $\lim\limits_{x\to\infty} e^{-x} = 0$

$\dfrac{1}{e^x}$

A. 0
B. 1
C. -1
D. e
E. ∞
F. $-\infty$

NO PARTIAL CREDIT

(d) (5 pts) $\lim\limits_{x\to1} \dfrac{x^2-1}{x^2-x} = \lim\limits_{x\to1} \dfrac{\cancel{(x-1)}(x+1)}{x\cancel{(x-1)}} = \dfrac{2}{1}$

A. DNE
B. 0
C. 1
D. 1.5
E. 2
F. ∞

2. (6 pts) Find the value of the constant k that makes the function $f(x)$ continuous everywhere.

$$f(x) = \begin{cases} -3x^2 + k & \text{for } x \leq -1 \\ 5x+6 & \text{for } x > -1 \end{cases}$$

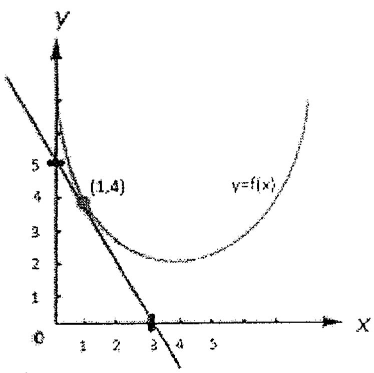
$-3(-1)^2 + k = 5(-1)+6 \leftarrow$ 4pts

$-3 + k = -5+6$ 2pts

$k = 3+1 = 4$

$\boxed{k = \quad 4}$

3. (6 pts) The parabola in the picture below is the graph of a function $f(x)$, and the line in the picture is the line tangent to the graph $y = f(x)$ at the point $(1,4)$. Find the value of $f'(1)$.



$f'(1) = \text{slope} \leftarrow$ 3pts

$= \dfrac{5-0}{0-3} = -\dfrac{5}{3}$

3pts

$\boxed{f'(1) = -\dfrac{5}{3}}$

4. (8 pts) Use the limit DEFINITION of the derivative to show that the derivative of the function $f(x) = 3 - 2x^2$ is $f'(x) = -4x$.

4pts

$f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{3-2(x+h)^2-(3-2x^2)}{h}$

4pts

$= \lim\limits_{h\to 0} \dfrac{\cancel{3}-2(x^2+h^2+2xh)-\cancel{3}+2x^2}{h} = \lim\limits_{h\to 0} \dfrac{-\cancel{2x^2}-2h^2-4xh+\cancel{2x^2}}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{h}(-2h-4x)}{\cancel{h}} = \lim\limits_{h\to 0} -2h-4x = -4x$

5. Compute the derivatives of the following functions; do not simplify the answer.

(a) (6 pts) $f(x) = \sin^2(3x)$

$$\boxed{f'(x) = 2\sin(3x)\cdot\cos(3x)\cdot 3}$$

(b) (6 pts) $g(x) = (1+3x^2)e^{5x}$

$$\boxed{g'(x) = 6x\cdot e^{5x} + (1+3x^2)e^{5x}\cdot 5}$$

(c) (6 pts) $h(x) = \sqrt{1+\tan x}$

$\frac{1}{2}(1+\tan x)^{-1/2}$ OK.

$$\boxed{h'(x) = \frac{1}{2\sqrt{1+\tan x}}\cdot \sec^2(x)}$$

6. (10 pts) Let $f(x) = \dfrac{x-1}{x+1}$. Compute $f'(2)$ and $f''(2)$.

$$f'(x) = \frac{1\cdot(x+1) - 1\cdot(x-1)}{(x+1)^2} = \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2}$$ (4pts)

$2(x+1)^{-2}$

$$f''(x) = 2(-2)(x+1)^{-3} = -\frac{4}{(x+1)^3}$$ (4pts)

$$f'(2) = \frac{2}{(2+1)^2} = \frac{2}{9}$$ (1pt)

$$f''(2) = -\frac{4}{(2+1)^3} = -\frac{4}{27}$$ (1pt)

$$\boxed{f'(2) = \frac{2}{9}}$$

$$\boxed{f''(2) = -\frac{4}{27}}$$

7. (10 pts) Find the derivative $\frac{dy}{dx}$ at the point with $x = -2$ and $y = 1$, given that

$$xy^2 - x^2 + y + 5 = 0$$

$$\frac{d}{dx}(xy^2 - x^2 + y + 5) = \frac{d}{dx}(0)$$

$$1\cdot y^2 + x\cdot 2y\frac{dy}{dx} - 2x + \frac{dy}{dx} + 0 = 0$$

(7pts)

$$\frac{dy}{dx}(2xy+1) = 2x - y^2$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=-2\\y=1}} = \frac{2(-2)-1^2}{2(-2)\cdot 1+1} = -5/3$$

(3pts)

$$\boxed{\left.\frac{dy}{dx}\right|_{(x,y)=(-2,1)} = -\frac{5}{3}}$$

8. (9 pts) Find the limit and provide a complete justification:

$$\lim_{x\to 0}\frac{e^{3x}+x-1}{\sin(x)} = \quad \frac{0}{0} \quad \text{L'H} \quad \lim_{x\to 0}\frac{\frac{d}{dx}(e^{3x}+x+1)}{\frac{d}{dx}(\sin x)} = \lim_{x\to 0}\frac{3\cdot e^{3x}+1}{\cos x} =$$

(6pts)

$$\frac{e^0+0-1}{\sin(0)} = \frac{1-1}{0} = \frac{0}{0}$$

$$= \frac{3e^0+1}{\cos 0} = \frac{3+1}{1} = 4$$

(3pts)

$$\boxed{4}$$

9. (5 pts) Let $f(x)$ be a function such that $f(1) = 5$ and $f'(1) = 3$. Use local linear approximation to find the approximate value of $f(1.2)$.

$$f(1.2) \simeq 5 + 3\cdot(1.2-1) \leftarrow \text{(4pts)}$$

$$= 5 + 3\cdot 0.2$$

$$= 5.6 \leftarrow \text{(1pt)}$$

$$\boxed{f(1.2) \simeq 5.6}$$

(The symbol $\simeq$ means "approximately equal to")

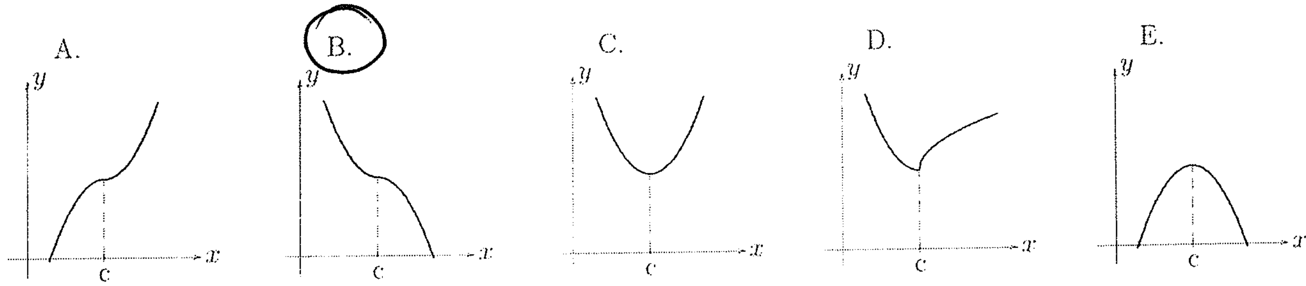
10. (7 pts) Suppose that a function has all of the following properties:

(a) $f''(x) > 0$ for $x < c$;

(b) $f'(c) = 0$

(c) $f'(x) < 0$ for $x > c$.

Which of the following could be the graph of f? (Circle the correct answer.)



NO PARTIAL CREDIT

11. (9 pts) Find the absolute minimum value of $f(x) = 3x^3 - x$ on the interval $\left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$; indicate the value of x where the minimum is achieved.

2pts $f'(x) = 9x^2 - 1$

2pts $f'(x) = 0 \rightsquigarrow 9x^2 - 1 = 0 \rightsquigarrow x^2 = \frac{1}{9}$, $x = \frac{1}{3}, -\frac{1}{3}$.

1pt $f\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - \frac{1}{3} = 3 \cdot \frac{1}{27} - \frac{1}{3} = \frac{1}{9} - \frac{1}{3} = \frac{1-3}{9} = -\frac{2}{9}$ min

1pt $f\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right) = 3\left(-\frac{1}{27}\right) + \frac{1}{3} = -\frac{1}{9} + \frac{1}{3} = \frac{-1+3}{9} = \frac{2}{9}$.

1pt $f\left(\frac{1}{\sqrt{3}}\right) = 3 \cdot \frac{1}{(\sqrt{3})^3} - \frac{1}{\sqrt{3}} = \cancel{3} \cdot \frac{1}{\cancel{3}\sqrt{3}} - \frac{1}{\sqrt{3}} = 0$

1pt $f\left(-\frac{1}{\sqrt{3}}\right) = 3 \cdot \left(-\frac{1}{\sqrt{3}}\right)^3 - \left(-\frac{1}{\sqrt{3}}\right) = -\cancel{3} \cdot \frac{1}{\cancel{3}\sqrt{3}} + \frac{1}{\sqrt{3}} = 0$.

All these need to be written, even if the answer is correct.

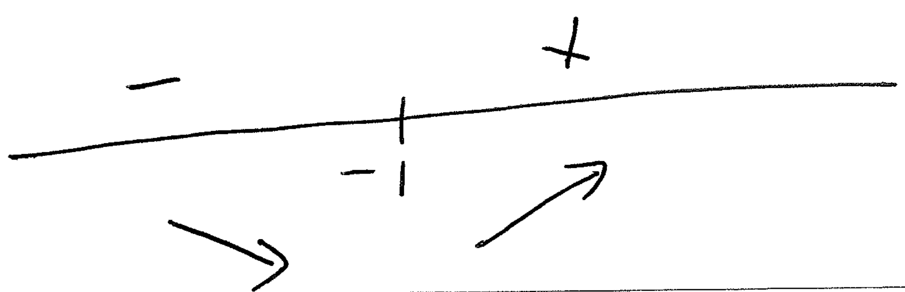
minimum value= $-\frac{2}{9}$ at $x = \frac{1}{3}$.

1pt

12. (9 pts) Consider the function $f(x) = \ln(x^2 + 2x + 5)$. Find the open intervals on which f is strictly increasing.

3pts $f'(x) = \dfrac{2x+2}{x^2+2x+5}$

2pts $f'(x) = 0 \rightsquigarrow 2x+2=0 \rightsquigarrow x=-1.$



4pts

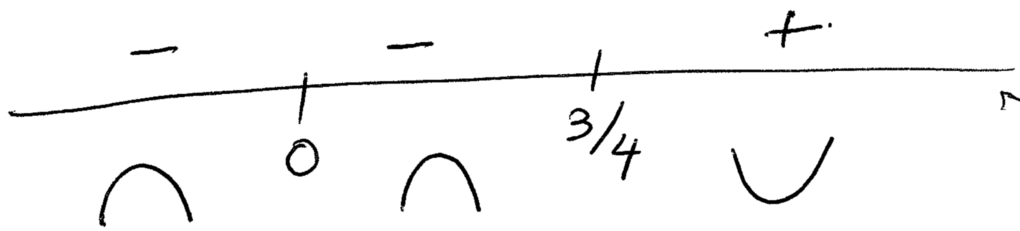
strictly increasing on: $(-1, \infty)$

13. (10 pts) Given the function $f(x) = 4x^5 - 5x^4$, find the open intervals on which f is concave up.

2pts $f'(x) = 20x^4 - 20x^3$

2pts $f''(x) = 20(4x^3 - 3x^2) = 20x^2(4x-3)$

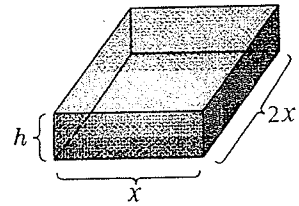
2pts $f''(x) = 0 \rightsquigarrow x=0,\ x=\frac{3}{4}$



4pts

concave up on: $(3/4, \infty)$

14. (10 pts) A rectangular cardboard box with NO top has a rectangular base, so that one side of the base is twice as long as the other. If the box is to have a volume of $\frac{4}{3}$ m^3, what should the height of the box be in order to *minimize* the amount of cardboard used?



1pt Volume: $2x^2h = \frac{4}{3}$

$h = \frac{2}{3x^2}$

$\text{Area} = x(2x) + 2(xh) + 2(2x)h = 2x^2 + 2xh + 4xh$ 2pts

$= 2x^2 + 6xh$

$= 2x^2 + 6x \cdot \frac{2}{3x^2} = 2x^2 + \frac{4}{x}$ 2pts

$A'(x) = 4x - \frac{4}{x^2}$ 2pts

$A'(x) = 0 \rightsquigarrow 4x = \frac{4}{x^2} \rightsquigarrow x^3 = 1 \rightsquigarrow x = 1$ 2pts

$\searrow \quad \nearrow$

$- \quad +$

1

$h = \frac{2}{3 \cdot 1^2} = \frac{2}{3}$ 1pt

height = $\frac{2}{3}$ m.

15. (~~8~~ 10 pts) A spherical balloon is inflated at a rate of 8 cubic centimeters per second. Find the rate at which the radius is increasing at the time when the radius is 5 centimeters.
(The volume of a sphere is $V = \frac{4}{3}\pi r^3$.)

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$ ← 6pts

$8 = \frac{4}{\cancel{3}}\pi \cancel{3}(5)^2 \frac{dr}{dt}$ ← 2pts

$\frac{dr}{dt} = \frac{8}{4\pi \cdot 25} = \frac{4}{25\pi}$ ← 2pts

rate= $\frac{4}{25\pi}$ cm/s

16. (8 pts) A particle moves along an s-axis. Use the given information to find the position function $s(t)$ of the particle, as a function of t.

$$v(t) = 2t - 3 \qquad s(1) = 3$$

$s(t) = \cancel{2}\frac{t^2}{\cancel{2}} - 3t + c$ ← 5pts

$S(1) = 1 - 3 + C = -2 + C$
$-2 + C = 3$ } 3pts

$C = 5$

1pt

$s(t) = t^2 - 3t + 5$

17. (10 pts) Evaluate the indefinite integral $\int \left(\pi - \frac{4}{x^3} + \frac{1}{\sqrt{x}}\right) dx$. Simplify the answer.

$\pi - 4x^{-3} + x^{-1/2}$

$I = \pi x - 4\frac{x^{-2}}{-2} + \frac{x^{1/2}}{1/2} + C = \pi x + \frac{2}{x^2} + 2\sqrt{x} + C$

3pts 3pts 3pts 1pt

$\pi x + \frac{2}{x^2} + 2\sqrt{x} + C.$

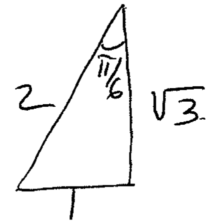
18. (10 pts) Evaluate the indefinite integral $I = \int \frac{e^x}{1 + e^x} dx$ using a u-substitution; show all work.

$u = e^x + 1$ 2pts
$du = e^x dx$ 2pts

$I = \int \frac{du}{u} = \ln u + c$ 4pts
$= \ln(1 + e^x) + c$ 2pts

$I = \ln(1 + e^x) + c$

19. (9 pts) Evaluate the definite integral $I = \int_0^{\frac{1}{6}} 4\cos(\pi x)\,dx$.



3pts

$$I = \frac{4\sin(\pi x)}{\pi}\Bigg|_0^{1/6} = \frac{4}{\pi}\left(\sin\frac{\pi}{6} - \sin 0\right)$$ ← 2pts

2 pts

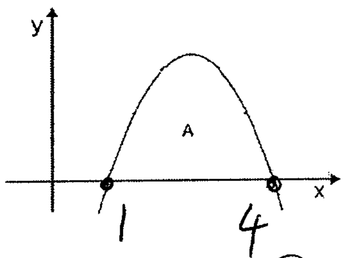
OK if the substitution is not spelled out.

$$= \frac{4}{\pi}\cdot\frac{1}{2} = \frac{2}{\pi}$$

2pts

$I = \frac{2}{\pi}$

20. (10 pts) Find the area A between the parabola $y = -x^2 + 5x - 4$ and the x-axis.



$$-x^2 + 5x - 4 = 0$$
$$x^2 - 5x + 4 = 0$$
$$(x-1)(x-4) = 0 \rightsquigarrow x = 1, 4$$

4 ← 1pt

2pts

$$A = \int_1^4 -x^2 + 5x - 4\,dx = -\frac{x^3}{3} + 5\frac{x^2}{2} - 4x\Bigg|_1^4 =$$

1 ← 1pt

$$= -\frac{4^3}{3} + 5\cdot\cancel{\frac{4^2}{2}}^{8} - 4\cdot 4 - \left(-\frac{1}{3} + \frac{5}{2} - 4\right) = -\frac{64}{3} + 40 - 16 + \frac{1}{3} - \frac{5}{2} + 4$$

$$= -\frac{63}{3} - \frac{5}{2} + 28 = -21 + 28 - \frac{5}{2} = 7 - \frac{5}{2} = \frac{9}{2}$$

AREA= $\frac{9}{2}$

21. (6 pts) If $\int_0^5 f(x)\,dx = 10$ and $\int_{-1}^0 f(x)\,dx = -7$, compute $I = \int_{-1}^5 f(x)\,dx$.

$I=$

19. (9 pts) Evaluate the definite integral $I = \int_0^{\frac{1}{6}} 4\cos(\pi x)\,dx$.

$$I = \underbrace{4\sin(\pi x)}_{3\text{pts}} \cdot \underbrace{\frac{1}{\pi}}_{2\text{pts}} \Big|_0^{1/6} = \underbrace{\frac{4}{\pi}\left(\sin\frac{\pi}{6} - \sin 0\right)}_{2\text{pts}} = \frac{4}{\pi}\left(\frac{1}{2} - 0\right)$$

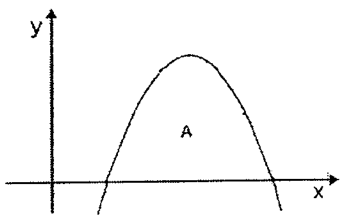
OK if the substitution is not spelled out.

$$= \frac{2}{\pi}$$

2pts

$\boxed{I = \frac{2}{\pi}}$

20. (10 pts) Find the area A between the parabola $y = -x^2 + 5x - 4$ and the x-axis.



$$-x^2 + 5x - 4 = 0$$

$$x^2 - 5x + 4 = 0$$

$$(x-1)(x-4) = 0 \rightsquigarrow x = 1, 4$$

$$A = \int_1^4 \underbrace{-x^2 + 5x - 4\,dx}_{2\text{pts}} = \underbrace{-\frac{x^3}{3} + 5\frac{x^2}{2} - 4x}_{3\text{pts}} \Big|_1^4$$

2pts

3pts

$$= -\frac{4^3}{3} + 5\cdot\frac{4^2}{2} - 4\cdot 4 - \left(-\frac{1}{3} + \frac{5}{2} - 4\right) = -\frac{64}{3} + 40 - 16 + \frac{1}{3} - \frac{5}{2} + 4$$

$$= -\frac{63}{3} - \frac{5}{2} + 28 = -21 + 28 - \frac{5}{2} = 7 - \frac{5}{2} = \frac{9}{2}$$

$\boxed{\text{AREA} = \frac{9}{2}}$

21. (6 pts) If $\int_0^5 f(x)\,dx = 10$ and $\int_{-1}^0 f(x)\,dx = -7$, compute $I = \int_{-1}^5 f(x)\,dx$.

$$10 - 7 = 3$$

$\boxed{I = 3}$