Name: ____________________

Instructor: ____________________

Time of your class: __________

UMKC Department of Mathematics and Statistics

Math 210 CALCULUS I
Common Final Examination

Saturday, May 6, 2017

INSTRUCTIONS:

1. Write your name, the name of your instructor and the time your class meets on top.
2. Only scientific calculators with no integration capabilities may be used for this examination. Graphing calculators and electronic communication devices are NOT permitted in the room.
3. Make sure that your test contains all problems indicated below on this cover page.
4. On page 1 ONLY: Multiple choice problems; full credit will be given for a correct answer and no credit for an incorrect answer; no work needs to be shown.
5. On all other pages: **All work MUST be shown to receive full credit.**
6. Unless specifically asked not to simplify, all answers need to be simplified completely.
7. EXACT answers are expected, NO approximations (unless otherwise specified).
8. Write your final answer in the box provided.

Problem	Possible	Earned
1	20	
2	7	
3	8	
4	8	
5	20	
6	7	
7	8	
8	8	
9	8	
10	10	
11	6	
12	10	
13	9	
14	10	
15	8	
16	10	
17	9	
18	9	
19	6	
20	10	
21	9	
Total	200	

1. (20 pts) Determine the following limits. Choose "DNE" (Does Not Exist) as your answer only if your function has NO limit, finite or infinite. **Circle one of the indicated choices**. No work needs to be shown.

(a) (5 pts) $\lim_{x\to-\infty} \frac{x-1+2x^3}{3x^3+4} =$

A. 0

B. DNE

C. $\frac{1}{3}$

D. $-\infty$

E. $\frac{2}{3}$

F. $-\frac{1}{4}$

(b) (5 pts) $\lim_{x\to1^+} \frac{x-5}{x-1} =$

A. 1

B. 5

C. -4

D. DNE

E. ∞

F. $-\infty$

(c) (5 pts) $\lim_{x\to\infty} e^{-x} =$

A. 0

B. 1

C. $\frac{1}{e}$

D. e

E. ∞

F. $-\infty$

(d) (5 pts) $\lim_{x\to\infty} \frac{2x\sqrt{x}+3}{x^2+1} =$

A. 0

B. ∞

C. 1

D. 2

E. DNE

F. 3

2. (7 pts) Consider the function:

$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ k & \text{if } x = 3 \end{cases}$$

Find the value of k such that the function $f(x)$ is continuous at $x = 3$.

$k =$

3. (8 pts) Let $f(x) = x^4$. Write the equation of the line tangent to the graph $y = f(x)$ at the point with x-coordinate $x = -1$.

$y =$

4. (8 pts) Use the limit DEFINITION of the derivative to show that the derivative of the function $f(x) = 4x^2 - x$ is $f'(x) = 8x - 1$.

5. (20 pts) Compute the derivatives of the following functions. Do NOT simplify.

(a) (6 pts) $f(x) = 3e^{x^2} + 7\ln(x^2+1)$

$f'(x) =$

(b) (4 pts) $g(x) = \sqrt{\sin x}$

$g'(x) =$

(c) (5 pts) $h(x) = \dfrac{\tan x}{x^2+1}$

$h'(x) =$

(d) (5 pts) $j(x) = x^3 \tan^{-1}(x)$

$j'(x) =$

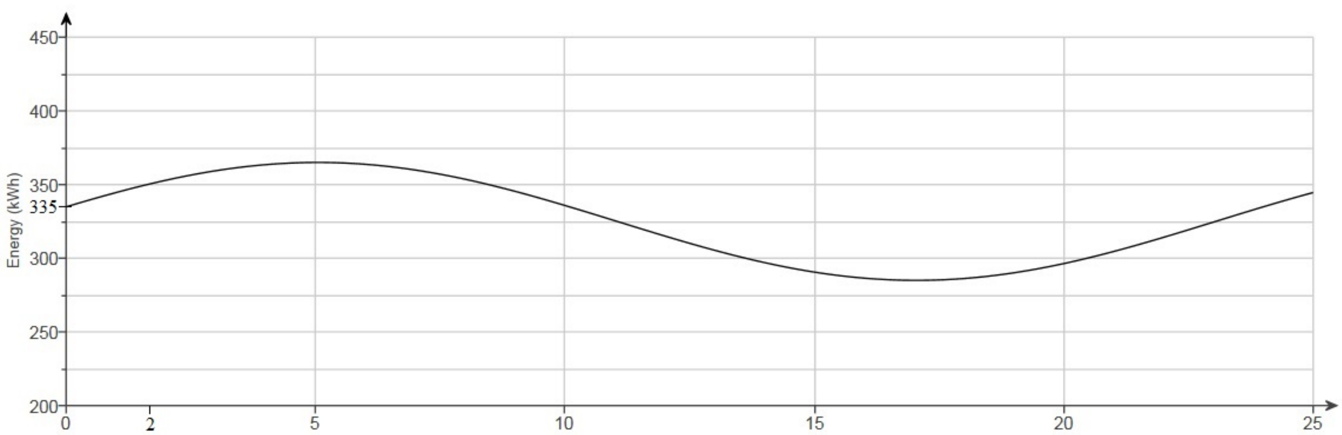
6. (7 pts) Find the linear approximation of $f(x) = \sqrt[3]{x}$ at $x = 1$ and use it to approximate $\sqrt[3]{2}$.

$\sqrt[3]{2} \simeq$

7. (8 pts) Use implicit differentiation to compute $\frac{dy}{dx}$, given the equation $x^3 + y^3 = 2xy$.

$\frac{dy}{dx} =$

8. (8 pts) Energy is the capacity to do work and power is the rate at which energy is consumed. Therefore, if $E(t)$ is the energy function for a system, then its derivative $P(t) = E'(t)$ is the power function. The graph below shows the energy consumed by a small community over a 25-hr period.



(a) (4 pts) At what times on the interval $[0, 25]$ is the power zero?

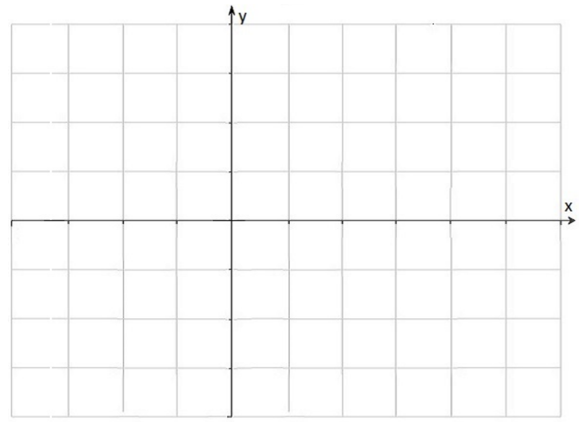
The power is zero at the times $t =$

(b) (4 pts) Estimate the power at $t = 2$ hr; show your work.

The power at $t = 2$ hr is approximately

9. (8 pts) Sketch the graph of a function that satisfies all of the following properties:

(a) $f(x)$ is continuous everywhere;
(b) $f'(x) < 0$ and $f''(x) < 0$ when $x < 0$;
(c) $f'(x) > 0$ and $f''(x) > 0$ when $0 < x < 2$;
(d) $f'(x) > 0$ and $f''(x) < 0$ when $x > 2$.



10. (10 pts) Use L'Hôpital's Rule to compute the limits below. Show all work.

(a) (5 pts) $\lim_{x \to 0} \frac{\ln(1+\pi x)}{x} =$

(b) (5 pts) $\lim_{x \to \frac{\pi}{3}} \frac{x - \frac{\pi}{3}}{\cos(x) - \frac{1}{2}} =$

11. (6 pts) The first derivative of a function has been computed to be $f'(x) = x^2(x-3)(x+4)$. Find the intervals where f is increasing or decreasing.

f is increasing on the open intervals:

f is decreasing on the open intervals:

12. (10 pts) Consider the function $f(x) = xe^{3x}$. This function has only one critical point.

(a) (6 pts) Find the x-coordinate of the critical point.

critical point at $x =$

(b) (3 pts) Circle the correct option below and show enough work to justify your answer.

A. The critical point is a point of local maximum.

B. The critical point is a point of local minimum.

C. The critical point is neither a local maximum, nor a local minimum.

(c) (1 pts) Is an **absolute** extreme value attained at the critical point? Explain why or why not.

13. (9 pts) Given the function $f(x) = 2x^4 - 3x^3 + 3$, find the open intervals on which f is concave up/concave down and the inflection point(s).

inflection point(s) at $x =$

concave up on the open interval(s):

concave down on the open interval(s):

14. (10 pts) A storage shed is to be built in the shape of a box with a square base. It is to have a volume of 891 cubic feet. The concrete for the base costs \$3 per square foot, the material for the roof costs \$8 per square foot, and the material for the sides costs \$4.50 per square foot. Find the dimensions of the most economical shed.

When setting up the cost function, use x to represent the length of one of the sides of the base.

side length=

height =

15. (8 pts) A spherical baloon is inflated with helium at a rate of $72\pi\ \frac{\text{ft}^3}{\text{min}}$. How fast is the balloon's radius increasing at the instant the radius is 3 ft?

(The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

16. (10 pts) Compute the integral: $I = \int \left(\frac{5x^2 - 12x^3}{x^3} + 5\sqrt[8]{x} \right) dx.$

$I =$

17. (9 pts) Use a change of variable (u-substitution) to compute the integral $I = \int \frac{\cos x}{(1 + 2\sin x)^3}\, dx.$

$I =$

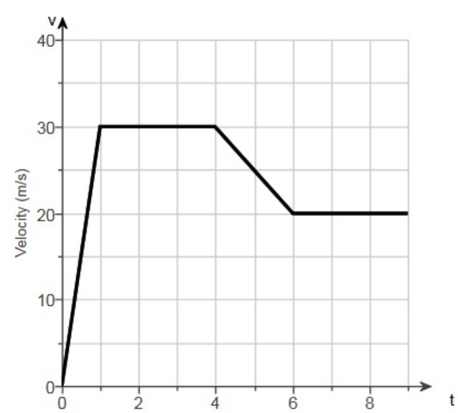
18. (9 pts) Consider the velocity function for an object moving along a line shown in the figure below.

(a) (6 pts) Complete the sentences by writing either "speeding up" or "slowing down" or "moving at a constant rate", as appropriate:

The object is ______ on the interval [0,1].

The object is ______ on the interval [3,4].

The object is ______ on the interval [4,6].



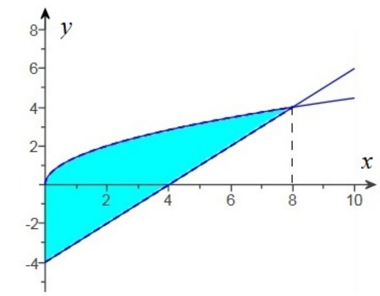
(b) (3 pts) Use geometry to find the displacement of the object between $t = 0$ and $t = 4$.

DISPLACEMENT=

19. (6 pts) Find the solution of the following initial value problem: $f'(x) = 10e^{-x}$; $f(0) = 100$.

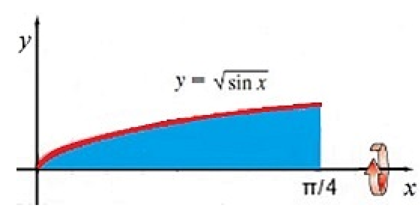
$f(x) =$

20. (10 pts) Find the area of the shaded region, between the graphs of $y = \sqrt{2x}$ and $y = x - 4$.



AREA=

21. (9 pts) Find the volume of the solid of revolution generated when the graph of $y = \sqrt{\sin x}$ and the x-axis from $x = 0$ to $x = \dfrac{\pi}{4}$ is rotated about the x-axis.



VOLUME=