Name: ______________________________

Instructor: ____________________________

Time of your class: __________

UMKC Department of Mathematics and Statistics

Math 210 CALCULUS I
Common Final Examination

Saturday, December 10, 2016

INSTRUCTIONS:

1. Write your name, the name of your instructor and the time your class meets on top.
2. Only scientific calculators with no integration capabilities may be used for this examination. Graphing calculators and electronic communication devices are NOT permitted in the room.
3. Make sure that your test contains all problems indicated below on this cover page.
4. On page 1 ONLY: Multiple choice problems; full credit will be given for a correct answer and no credit for an incorrect answer; no work needs to be shown.
5. On all other pages: **All work MUST be shown to receive full credit.**
6. Unless specifically asked not to simplify, all answers need to be simplified completely.
7. EXACT answers are expected, NO approximations (unless otherwise specified).
8. Write your final answer in the box provided.

Problem	Possible	Earned
1	20	
2	5	
3	8	
4	8	
5	20	
6	5	
7	8	
8	6	
9	9	
10	8	
11	12	
12	8	
13	10	
14	10	
15	9	
16	10	
17	6	
18	9	
19	8	
20	12	
21	9	
Total	200	

1. (20 pts) Determine the following limits. Choose "DNE" (Does Not Exist) as your answer only if your function has NO limit, finite or infinite. **Circle one of the indicated choices**. No work needs to be shown.

(a) (5 pts) $\lim_{x \to -\infty} \dfrac{2x^4 + 5x - 1}{3x^3 + 4} =$

A. DNE

B. ∞

C. $-\infty$

D. $\dfrac{2}{3}$

E. $-\dfrac{1}{4}$

F. 0

(b) (5 pts) $\lim_{x \to 1^-} \dfrac{x+2}{x-1} =$

A. 1

B. -2

C. $-\dfrac{1}{2}$

D. DNE

E. ∞

F. $-\infty$

(c) (5 pts) $\lim_{x \to 1} \dfrac{\sin(x-1)}{x^2 - 1} =$

A. 0

B. 1

C. -1

D. $\dfrac{1}{2}$

E. 2

F. ∞

(d) (5 pts) $\lim_{x \to \infty} \dfrac{\sqrt{4x^2 + 5}}{2x^2 + 1} =$

A. 0

B. ∞

C. 1

D. 2

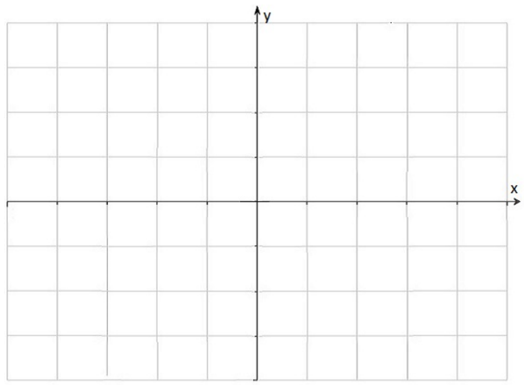
E. DNE

F. 5

2. (5 pts) Draw below the graph of a function defined on $(-\infty, \infty)$ such that the following conditions are satisfied:

(a) The limits $\lim_{x \to 1^+} f(x)$ and $\lim_{x \to 1^-} f(x)$ both exist, but $\lim_{x \to 1} f(x)$ does not exist.

(b) $f(x)$ is continuous but not differentiable at $x = -2$.



3. (8 pts) Let $f(x) = x^3$. Write the equation of the line tangent to the graph $y = f(x)$ at the point with x-coordinate $x = 2$.

$y =$

4. (8 pts) Use the limit DEFINITION of the derivative to show that the derivative of the function $f(x) = 2 - 3x^2$ is $f'(x) = -6x$.

5. Compute the derivatives of the following functions. Do NOT simplify.

(a) (5 pts) $f(x) = \sqrt{x^5 + 2^x}$

$f'(x) =$

(b) (5 pts) $g(x) = \sin(x^2) - \sin^2(x)$

$g'(x) =$

(c) (5 pts) $h(x) = \dfrac{e^{3x}}{e^x + 1}$

$h'(x) =$

(d) (5 pts) $j(x) = x^3 \tan(x)$

$j'(x) =$

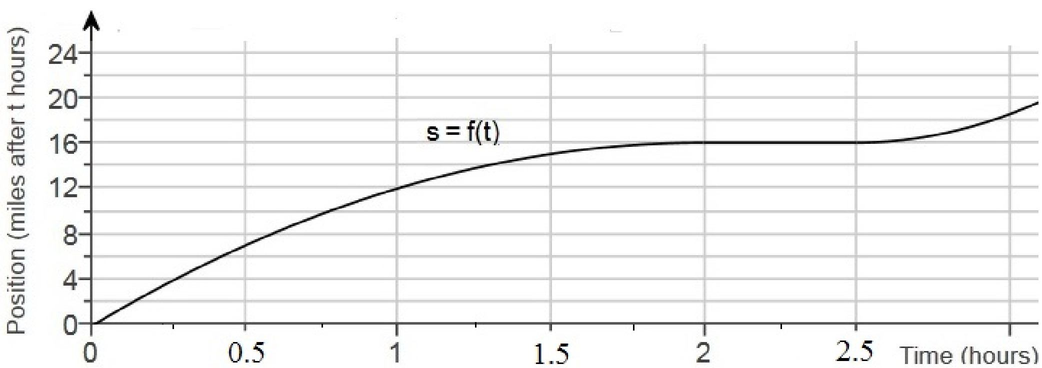
6. (5 pts) Use logarithmic diffentiation to compute $\dfrac{dy}{dx}$ for $y = x^x$. Show all work.

$\dfrac{dy}{dx} =$

7. (8 pts) Use implicit differentiation to find the derivative $\frac{dy}{dx}$, given that $y^2 - 3xy + x^2 = 1$.

$\frac{dy}{dx} =$

8. (6 pts) The following graph shows the position s of a bicyclist after t hours from a start time.



(a) (2 pts) Does the bicyclist speed up or slow down during the first two hours? Explain.

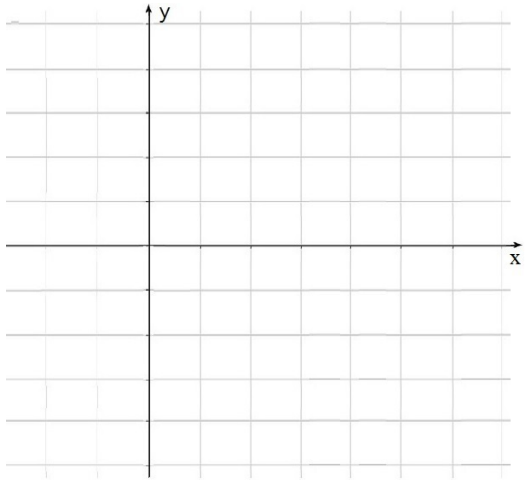
(b) Based on the information in the graph, appoximate as accurately as possible the following velocities. Explain clearly how the answer has been obtained.

i. (2 pts) The (instantaneous) velocity when $t = 1$;

ii. (2 pts) The (instantaneous) velocity when $t = 2.2$.

9. (9 pts) Sketch the graph of a function with the following properties:

(a) The domain of $f(x)$ is the interval $(0,\infty)$ and $f(x)$ has vertical asymptote at $x=0$.
(b) $f'(x)>0$ on the interval $(0,\infty)$.
(c) $f''(x)<0$ for all x on the interval $(0,3)$ and $f''(x)>0$ on the interval $(3,\infty)$.



10. (8 pts) Let $f(x)=x+\dfrac{2}{x}$. Find the absolute minimum and the absolute maximum value of $f(x)$ on the interval $[1,3]$.

absolute minimum value =   at $x =$

absolute maximum value =   at $x =$

11. (12 pts) Use L'Hôpital's Rule to compute the limits below. Show all work.

(a) (6 pts) $\displaystyle\lim_{x\to 3}\frac{e^x-e^3}{\sin(x-3)}=$

(b) (6 pts) $\displaystyle\lim_{x\to\infty}\frac{\sqrt{x}}{\ln x}=$

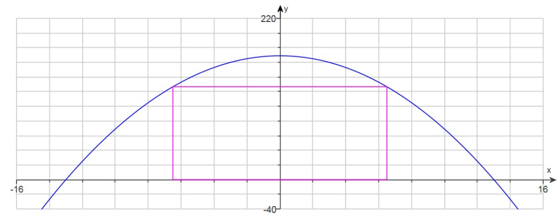
12. (8 pts) The first derivative of a function $f(x)$ has been computed to be $f'(x) = \frac{(x-2)^2(x+1)}{x-3}$. Find the open intervals where $f(x)$ is increasing/decreasing and find the x-coordinates of the points of local (relative) maximum/minimum, if any.

$f(x)$ is increasing on:
$f(x)$ is decreasing on:
point(s) of local (relative) minimum at $x =$
point(s) of local (relative) maximum at $x =$

13. (10 pts) Given the function $f(x) = \frac{1}{6}x^6 - 5x^4$, find the open intervals on which f is concave up/concave down and the inflection point(s).

concave down on:
concave up on:
inflection point(s) at $x =$

14. (10 pts) A rectangle is constructed with its base on the x-axis and two of its vertices on the parabola $y = 169 - x^2$. What are the dimensions (length and width) of the rectangle with the maximum area? Find the maximum area.

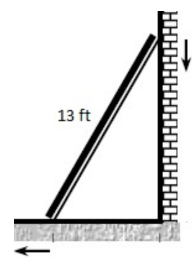


length =

width =

maximum area =

15. (9 pts) A 13 ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/s, how fast will the foot of the ladder be moving away from the wall when the top is 5 ft above the ground?



16. (10 pts) Compute the integral: $I = \int \frac{x^3+1}{x} + \frac{1}{x^2+1} + 5\sqrt{x}\,dx$

$I =$

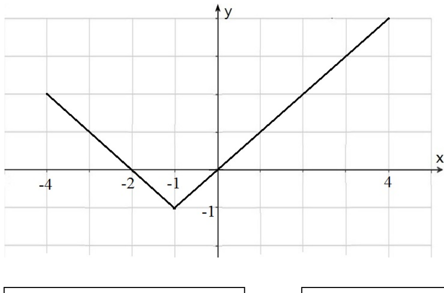
17. (6 pts) Compute the integral $I = \int_0^{\frac{\pi}{6}} \sin(x)\,dx$

$I =$

18. (9 pts) Use a change of variable (u-substitution) to compute the integral $I = \int \frac{x^2}{(5x^3+1)^6}\,dx$.

$I =$

19. (8 pts) Consider the function $f(x)$ whose graph is given below. Use geometry to compute the following integrals:



$\int_0^4 f(x)dx =$	$\int_{-2}^0 f(x)dx =$	$\int_{-4}^{-2} f(x)dx =$	$\int_{-4}^4 f(x)dx =$

20. (12 pts) The velocity of an object moving along a line is given by $v(t) = 3t^2 - 12$, for $0 \leq t \leq 5$.

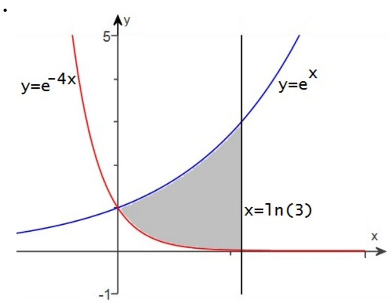
(a) (6 pts) Find the displacement of the object during the given time interval.

displacement =

(b) (6 pts) Find the distance travelled during the given time interval.

distance travelled =

21. (9 pts) Find the area of the region bounded by $y = e^x$, $y = e^{-4x}$ and $x = \ln(3)$.



AREA=