

Name: _____

Instructor: _____

Time of your class: _____

UMKC Department of Mathematics and Statistics

Math 210 CALCULUS I
Common Final Examination
Saturday, May 10, 2014

INSTRUCTIONS:

1. Write your name, the name of your instructor and the time your class meets on top.
2. Only scientific calculators may be used for this examination. Graphing calculators and electronic communication devices are NOT permitted in the room.
3. Make sure that your test contains all problems indicated below on this cover page.
4. On page 1 ONLY: Multiple choice problems; full credit will be given for a correct answer and no credit for an incorrect answer; no work needs to be shown.
5. On all other pages: **All work MUST be shown to receive full credit.**
6. Unless specifically asked not to simplify, all answers need to be simplified completely.
7. EXACT answers are expected, NO approximations (unless otherwise specified).
8. Write your final answer in the box provided.

Problem	Possible	Earned
1	15	
2	9	
3	9	
4	9	
5	5	
6	11	
7	8	
8	9	
9	16	
10	8	
11	10	
12	9	
13	8	
14	9	
15	9	
16	9	
17	8	
18	9	
19	5	
20	6	
21	10	
22	9	
Total	200	

1. (15 pts) Determine the following limits. Choose “DNE” (Does Not Exist) as your answer only if your function has NO limit, finite or infinite. **Circle one of the indicated choices.** No work needs to be shown.

(a) (5 pts) $\lim_{x \rightarrow \infty} \frac{-x^4 + 3x - 1}{x^3 - 2x + 1} =$

- A. 1
- B. 0
- C. -1
- D. DNE
- E. ∞
- F. $-\infty$

(b) (5 pts) $\lim_{x \rightarrow 3^+} \frac{x}{x - 3} =$

- A. 0
- B. 1
- C. 3
- D. DNE
- E. ∞
- F. $-\infty$

(c) (5 pts) $\lim_{y \rightarrow 6} \frac{y^2 - 36}{y - 6} =$

- A. 1
- B. 6
- C. 12
- D. 36
- E. DNE
- F. ∞

2. (9 pts) Consider the function $f(x) = \begin{cases} x - 4 & \text{for } x \leq 0 \\ x^2 + 1 & \text{for } x > 0 \end{cases}$

(a) (6 pts) Find the following limits; write “DNE” if the limit is undefined.

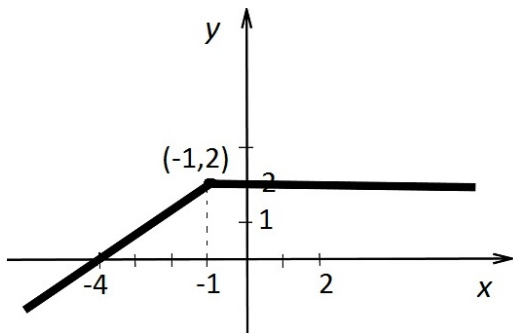
$$\lim_{x \rightarrow 0^-} f(x) =$$

$$\lim_{x \rightarrow 0^+} f(x) =$$

$$\lim_{x \rightarrow 3} f(x) =$$

(b) (3 pts) Is the function $f(x)$ above continuous for all values of x ? Why or why not?

3. (9 pts) The graph of a function $f(x)$ is sketched below.



Find the following derivatives. If a derivative does not exist, write “DNE”.

$$f'(-4) =$$

$$f'(-1) =$$

$$f'(2) =$$

4. (9 pts) Find the equation of the line tangent to the graph of $y = x^2$ at the point $(-3, 9)$.

$$y =$$

5. (5 pts) Recall that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

A spherical balloon is being inflated. Find the rate of change of the volume V with respect to the radius r at the instant when $r = 4$.

rate of change=

6. (11 pts) Find the derivatives of the following functions. Do NOT simplify your answer.

(a) (5 pts) $g(x) = \frac{x^3}{x^2 - 1}$

$g'(x) =$

(b) (6 pts) $h(x) = e^{3x} \ln(x^2 + 1)$

$h'(x) =$

7. (8 pts) Let $f(x) = \sqrt{3x + 1}$. Compute $f'(1)$ and $f''(1)$.

$f'(1) =$

$f''(1) =$

8. (9 pts) Find $\frac{dy}{dx}$ by differentiating implicitly: $x^6 - 5x^2y^2 + y^3 = 6$.

$$\frac{dy}{dx} =$$

9. (16 pts) Find the limits and justify completely the answers.

(a) (8 pts) $\lim_{x \rightarrow 1} \frac{\sin(\pi x)}{\ln x} =$



(b) (8 pts) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} =$



10. (8 pts) Consider the function $f(x) = x^2 - 3x$. Show, using the **DEFINITION** of the derivative, that $f'(x) = 2x - 3$.

11. (10 pts) The following information about a function $f(x)$ is known:

(a) $f(x)$ is defined and differentiable for all real x , except for $x = 2$.

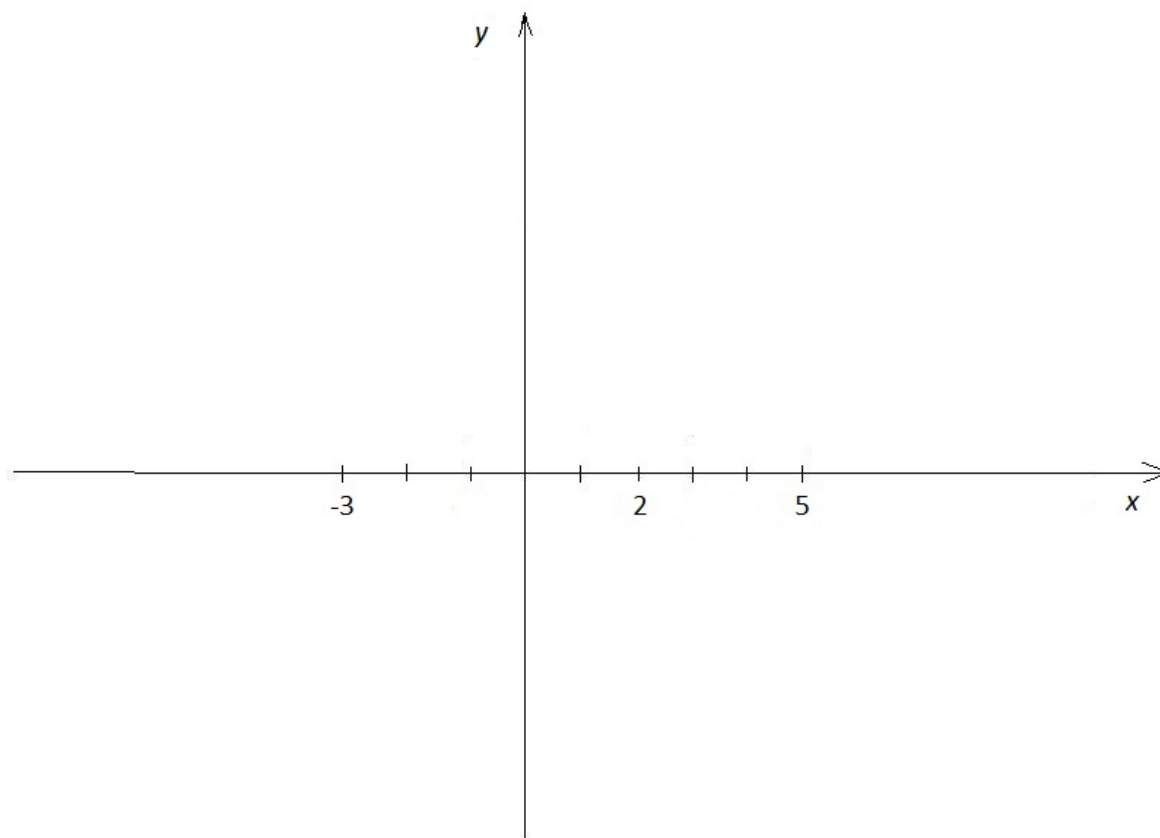
(b) $\lim_{x \rightarrow -\infty} f(x) = 0 = \lim_{x \rightarrow \infty} f(x)$.

(c) $\lim_{x \rightarrow 2^-} f(x) = \infty$ and $\lim_{x \rightarrow 2^+} f(x) = -\infty$.

(d) $f'(x) < 0$ for all $x < -3$ and $f'(x) < 0$ for all $x > 5$.

(e) $f'(-3) = 0 = f'(5)$

Sketch below the graph of a function that *clearly* satisfies all these conditions.



12. (9 pts) Consider the function $f(x) = x^3e^x$. Find all open intervals where f is increasing.

f is increasing on the interval(s):

13. (8 pts) The second derivative of a function $f(x)$ has been computed to be $f''(x) = x^2 - 2$. Find all open intervals where f is concave up.

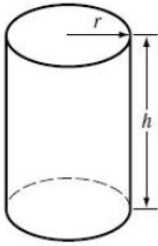
f is concave up on the interval(s):

14. (9 pts) Consider the function $f(x) = \sqrt{3}x - 2\sin(x)$, for $0 < x < 2\pi$. Find the points of relative extrema of $f(x)$.

relative maximum at $x =$

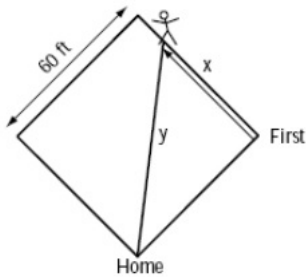
relative minimum at $x =$

15. (9 pts) A cylindrical can, open at the top, is to hold 510 cm^3 of liquid. Find the radius r that minimizes the amount of material needed to manufacture the can. (The surface area of the can is $S = \pi r^2 + 2\pi r h$ and the volume is $V = \pi r^2 h$.)



$r =$

16. (9 pts) A softball diamond is a square whose sides are 60 ft long. Suppose that a player running from first to second base has a speed of 25 ft/s at the instant when she is at a distance x of 20 ft from first base. At what rate is the player's distance y from home plate changing at that instant?



rate =

17. (8 pts) Evaluate the following indefinite integral using a substitution: $\int \cos^3 x \sin x \, dx$

$$\int \cos^3 x \sin x \, dx =$$

18. (9 pts) A particle moves with a velocity $v(t)$ m/s along an s -axis, where $v(t) = 1 + \frac{1}{t^2}$. Find the distance traveled by the particle during the time interval $1 \leq t \leq 2$.

distance=

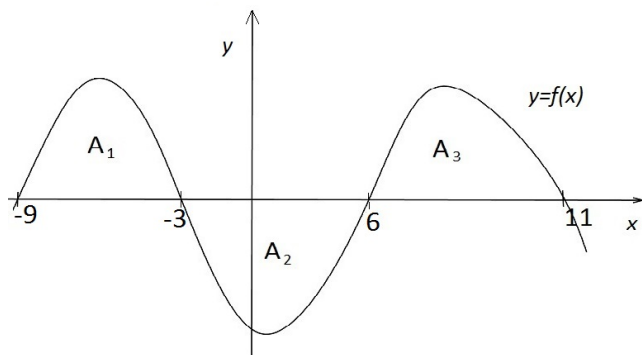
19. (5 pts) The graph of a function $f(x)$ is sketched below.

Let A_1 denote the area between $y = f(x)$ and the x -axis over the interval $[-9, -3]$.

Let A_2 denote the area between $y = f(x)$ and the x -axis over the interval $[-3, 6]$.

Let A_3 denote the area between $y = f(x)$ and the x -axis over the interval $[6, 11]$.

If $A_1 = 12$, $A_2 = 14$ and $A_3 = 15$, compute the integral $\int_{-9}^{11} f(x) \, dx$.

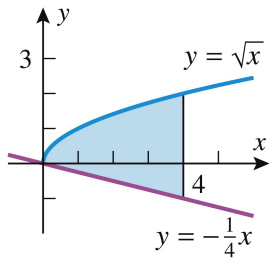


$$\int_{-9}^{11} f(x) \, dx =$$

20. (6 pts) Evaluate the integral $I = \int_0^{\ln 2} 5e^{3x} dx$.

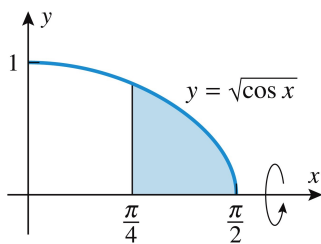
I=

21. (10 pts) Find the area of the shaded region:



AREA=

22. (9 pts) Find the volume of the solid that results when the shaded region below is revolved about the x -axis.



VOLUME=