Name: ______________________________

Instructor: ____________________________

Time of your class: __________

UMKC Department of Mathematics and Statistics

Math 210 CALCULUS I

Common Final Examination

Saturday , April 30th 2011

1. Write your name, the name of your instructor and the time your class meets on top of this page.
2. Only scientific calculators may be used for this examination. Graphing calculators and electronic communication devices are NOT permitted in the room.
3. Make sure that your test contains all problems indicated below on this cover page.
4. On page 1 ONLY: Multiple choice problems; full credit will be given for a correct answer and no credit for an incorrect answer; NO work needs to be shown.
5. On all other pages: All work MUST be shown to receive full credit.
6. Unless specifically asked not to simplify, all answers need to be simplified completely. **Exact answers are expected, NO approximations.**
7. Write your final answer in the box provided.

Problem	Possible	Earned
1	18	
2	6	
3	10	
4	14	
5	8	
6	8	
7	10	
8	16	
9	20	
10	10	
11	12	
12	8	
13	10	
14	10	
15	10	
16	10	
17	10	
18	10	
Total	200	

1. (18 pts) Determine the following limits. Choose "DNE" (Does Not Exist) as your answer only if your function has NO limit, finite or infinite. **Circle one of the indicated choices.** No work needs to be shown.

(a) (6 pts) If $f(x) = \begin{cases} 2 & \text{if } x = 0 \\ \dfrac{\sin x}{x} & \text{if } x \neq 0 \end{cases}$, then $\lim_{x \to 0} f(x) =$

A. 1

B. 2

C. DNE

D. ∞

E. $-\infty$

F. 0

(b) (6 pts) $\lim_{x \to -\infty} \dfrac{\sqrt{x^2+3}}{x+1} =$

A. 1

B. -1

C. DNE

D. ∞

E. $-\infty$

F. 0

(c) (6 pts) $\lim_{x \to 2^-} \dfrac{x-2}{x^2-4x+4} =$

A. 1

B. -1

C. DNE

D. ∞

E. $-\infty$

F. 0

2. (6 pts) Consider the function

$$f(x) = \begin{cases} 3x, & x < 1 \\ 1 + e^{kx}, & x \geq 1 \end{cases}$$

Find a value of the constant k, if possible, that will make the function $f(x)$ continuous everywhere.

$k =$

3. (10 pts) Consider the function $f(x) = 4\sqrt{x} - x^2 + 2$.

Write the equation of the line tangent to $y = f(x)$ at the point with x-coordinate $x = 4$. Use the slope-intercept form to give your final answer.

$y =$

4. (14 pts) Find the derivative of the following functions; do not simplify your answer.

(a) (8 pts) $f(x) = \cos(2x) \cdot \sin(x)$

$f'(x) =$

(b) (6 pts) $g(x) = (\ln x)^5$

$g'(x) =$

5. (8 pts) Compute the derivative of $h(x) = \dfrac{x^3 + x^2}{e^{3x}}$ and simplify your answer.

$h'(x) =$

6. (8 pts) If $y = (2x)^x$, find the derivative $\dfrac{dy}{dx}$. (Use logarithmic differentiation.)

$\dfrac{dy}{dx} =$

7. (10 pts) If $xy + 2x + y^2 = 10$, find $\dfrac{dy}{dx}$ at the point with $(x, y) = (3, 1)$.

$$\left.\frac{dy}{dx}\right|_{(x,y)=(3,1)} =$$

8. (16 pts) Find the limits and show all the work involved.

(a) (8 pts) $\displaystyle\lim_{x\to\infty} \frac{x^3}{e^x}$

(b) (8 pts) $\displaystyle\lim_{x\to\infty} \frac{\ln(3x^2+1)}{\ln(x)}$

9. (20 pts) Consider the function $f(x) = x^4 - 6x^2 + 5$.

(a) (8 pts) Find all open intervals where f is increasing.

f is increasing on the interval(s):

(b) (8 pts) Find all open intervals where f is concave up.

f is concave up on the interval(s):

(c) (4 pts) Find all values of x for which f has a local minimum.

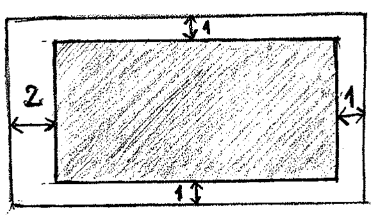
$x =$

10. (10 pts) A 13 ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/s, how fast will the foot of the ladder be moving away from the wall when the top is 5 ft above the ground?

rate=

11. (12 pts) A rectangular page is to contain 96 square inches of printable area. The margins at top and bottom are each 1 inch, the right side margin is 1 inch and the left side margin is 2 inches. What should the dimensions of the page be so that the least amount of paper is used?

(The printable area is the shaded portion in the picture below)



height=
width=

12. (8 pts) You are given the following information regarding a function $f(x)$:

(a) $f(x)$ is defined and continuous for all real x.

(b) $f(x)$ is differentiable everywhere, except at $x = 0$.

(c) $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 4$.

(d) The solutions of the equation $f'(x) = 0$ are $x = -4$ and $x = -2$.

Sketch below a graph of a function $f(x)$ that satisfies all of the above requirements, making sure that your graph *clearly* illustrates them.

13. (10 pts) Find the absolute minimum and the absolute maximum of the function

$$f(x) = x^3 e^x \quad \text{on the interval } [-4, 1]$$

absolute min=

absolute max=

14. (10 pts) Find the indefinite integral $\int \frac{5}{\sqrt{3x+7}}\,dx$.

15. (10 pts) A particle moves with velocity $v(t) = t^{2/3}$ along an s-axis. Let $s(t)$ denote the position function. If $s(8) = 0$, determine $s(t)$.

$s(t) =$

16. (10 pts) Compute the definite integral $I = \int_0^1 \frac{4t^2}{t^3+1}\,dt$.

$I=$

17. (10 pts) Find the total area between the graph of $y = \sin x$ and the x-axis over the interval $[0, \frac{4\pi}{3}]$.

(Start by sketching the graph of the function and identifying the area to be computed.)

AREA=

18. (10 pts) Show, **using the DEFINITION of the derivative**, that the derivative of $f(x) = \frac{1}{x}$ is $f'(x) = -\frac{1}{x^2}$.