

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

UMKC Department of Mathematics and Statistics

# Math 210 CALCULUS I

## Common Final Examination

Saturday , May 1st, 2010

11:00AM to 1:00PM

Only scientific calculators may be used for this examination. Graphing calculators and electronic communication devices are not permitted in the room. Make sure that your test contains all problems indicated below on this cover page.

All work *must* be shown to receive full credit.

Problem	Possible	Earned
1	8	
2	12	
3	10	
4	12	
5	24	
6	12	
7	10	
8	16	
9	12	
10	12	
11	24	
12	12	
13	16	
14	10	
15	10	
Total	200	

1. (8 pts) Let

$$g(t) = \begin{cases} t^2 + 1 & \text{for } t \leq 1 \\ 2 + \ln t & \text{for } t > 1. \end{cases}$$

Does the limit  $\lim_{t \rightarrow 1} g(t)$  exist? Prove your answer.

2. (12 pts) Find the limits:

a)  $\lim_{t \rightarrow +\infty} \frac{\sin t}{t}$

b)  $\lim_{y \rightarrow 3} \frac{y - 4}{(3 - y)^2}$

3. (10 pts) Use the definition of derivative and the properties of limits to find  $f'(x)$ , if  $f(x) = 3x^2 - x$ . (**Warning:** No other formulae for derivatives may be used in this problem.)

4. (12 pts) Find the second derivative of:

$$f(x) = 7x^2 - 2x + \ln x + 18x^{1/3} + 3 \cos x$$

5. (24 pts) Find the derivatives of the following functions. Do not simplify.

a)  $x^2e^x$

b)  $\frac{\sin x}{x^2 + 3}$

c)  $[x^4 + \sin(5x^2 + 1)]^{-3}$

d)  $\ln(\tan x)$

6. (12 pts) Write the equation of the line tangent to the curve  $3x^3 + y^3 = 10xy$  at the point  $(1, 3)$ .

7. (10 pts) Find the limit:

$$\lim_{t \rightarrow 0} \frac{2te^t}{1 - e^t}$$

8. (16 pts) Sketch the graph of a function  $f$  which is continuous for  $x \neq 0$  and satisfies the following conditions:

$$\lim_{x \rightarrow -\infty} f(x) = 3, \quad \lim_{x \rightarrow 0^-} f(x) = -\infty,$$

$$\lim_{x \rightarrow +\infty} f(x) = 1, \quad \lim_{x \rightarrow 0^+} f(x) = 2,$$

$$f(1) = 4, \quad f(0) = 0$$

Interval	Sign of $f'(x)$	Sign of $f''(x)$
$(-\infty, 0)$	—	—
$(0, 1)$	+	—
$(1, 2)$	—	—
$(2, \infty)$	—	+

9. (12 pts) A rectangular container with two square sides and an open top is to have a volume of 36 cubic feet. Find the dimensions of the container with minimum surface area.

10. (12 pts) Find the interval(s) where the function  $f(x) = xe^{-x^2}$  is increasing.

11. (24 pts) Find the indefinite integrals:

$$\text{a) } \int \frac{5x^4 - 3\sqrt{x} + 2}{x} dx$$

$$\text{b) } \int \left( \frac{4}{1+x^2} + \frac{1}{e^{3x}} \right) dx$$

$$\text{c) } \int \frac{\cos x}{\sin^3 x} dx$$



12. (12 pts) Find the area bounded by  $y = x^2 - 1$  and  $y = 2x + 2$ .

13. (16 pts) Evaluate the following definite integrals

a)  $\int_{-1}^2 f(x) dx$ , where  $f(x) = \begin{cases} 3 & \text{for } -1 < x < 1 \\ 4 - x^2 & \text{for } 1 < x < 2. \end{cases}$

b)  $\int_{-\sqrt{\pi}}^0 3x \sin(x^2) dx$

14. (10 pts) Sketch the region  $B$  in the first quadrant bounded by the curves  $y = x^3$  and  $x = y$ . Set up a definite integral for the volume of the solid obtained by rotating  $B$  around the  $y$ -axis. DO NOT evaluate the integral.

15. ( 10 pts) Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of  $6 \text{ mi}^2/h$ . How fast is the radius of the spill increasing when the area is  $9 \text{ mi}^2$ .