Name: _____

Instructor: _____

UMKC Department of Mathematics and Statistics

Math 210 CALCULUS I
Common Final Examination

Saturday , May 2, 2009
11:00AM to 1:00PM

Only scientific calculators may be used for this examination. Graphing calculators and electronic communication devices are not permitted in the room. Make sure that your test contains all problems indicated below on this cover page.

All work *must* be shown to receive full credit.

Problem	Possible	Earned
1	8	
2	12	
3	10	
4	12	
5	24	
6	12	
7	10	
8	16	
9	12	
10	12	
11	24	
12	12	
13	16	
14	10	
15	10	
Total	200	

1. (8 pts) Let

$$g(t) = \begin{cases} t^2 - 1 & \text{for } t \leq 1 \\ 1 - \cos t & \text{for } t > 1. \end{cases}$$

Does the limit $\lim_{t \to 1} g(t)$ exist? Prove your answer.

2. (12 pts) Find the limits (if they exist):

a) $\lim_{x \to -\infty} e^{\frac{1}{x}}$

b) $\lim_{x \to 0^-} \dfrac{\sin x}{x^2}$

3. (10 pts) Use the definition of derivative and the properties of limits to find $f'(x)$, if $f(x) = 3x^{-1}$. (**Warning**: No other formulae for derivatives may be used in this problem.)

4. (12 pts) Find the second derivative of:

$$f(x) = 5x^2 - 7x + 2\ln x + 16x^{3/4} - 4\cos x$$

5. (24 pts) Find the derivatives of the following functions. Do not simplify.

a) $x^2 \cos x$

b) $\dfrac{\sin x}{x^2 + 3}$

c) $(x^3 + e^{4x})^{-2}$

d) $\cot(\ln(x^2 + 3))$

6. (12 pts) Write the equation of the line tangent to the curve $\sqrt{y} + xy^2 = 5$ at the point $(4, 1)$.

7. (10 pts) Find the limit:

$$\lim_{x \to 0} \frac{\ln(e^x + x)}{x}$$

8. (16 pts) Sketch the graph of a function f which is continuous for $x \neq 1$ and satisfies the following conditions:

$$\lim_{x\to-\infty} f(x) = 2, \qquad \lim_{x\to1^-} f(x) = +\infty,$$
$$\lim_{x\to+\infty} f(x) = 1, \qquad \lim_{x\to1^+} f(x) = 3$$

Interval	Sign of $f'(x)$	Sign of $f''(x)$
$(-\infty, 1)$	$+$	$+$
$(1, 2)$	$+$	$-$
$(2, 4)$	$-$	$-$
$(4, \infty)$	$-$	$+$

9. (12 pts) An open box is to be made from a 3-ft by 8-ft rectangular piece of sheet metal by cutting out squares of equal size from the four corners and bending up the sides. Find the size of the square that must be cut from the corners to make the box of maximum volume. Make sure to explain why your answer will do it.

10. (12 pts) Find the interval(s) where the function $f(x) = x^3 \ln x$ is decreasing.

11. (24 pts) Find the indefinite integrals:

a) $\displaystyle\int \frac{5x^4 - 3x + 2}{x^2}\, dx$

b) $\displaystyle\int \left(\frac{4}{1 + 4x^2} + \frac{1}{e^{5x}} \right) dx$

c) $\displaystyle\int \frac{\sin x}{\cos^2 x}\ dx$

12. (12 pts) Find the area of the region bounded by the curves $y = x^2 - 2$ and $y = 2x + 1$.

13. (16 pts) Evaluate the following definite integrals

a) $\displaystyle\int_{-1}^{2} f(x)\,dx,$ where $f(x) = \begin{cases} x & \text{for } -1 < x < 1 \\ 2 - x^2 & \text{for } 1 < x < 2. \end{cases}$

b) $\displaystyle\int_{0}^{\sqrt{\pi}} x\cos\left(x^2\right)dx$

14. (10 pts) Find the volume of the solid whose base is the region in the first quadrant bounded by the curves $y = x^2$ and $x = y$, and whose cross sections perpendicular to the x-axis are squares.

15. (10 pts) Air is being pumped into a spherical baloon at the constant rate of 40 cm^3 per second. Determine the rate at which the radius of the baloon is increasing when the diameter of the baloon is 10 cm.