Name: ______________________________

Instructor: ____________________________

Time of your class: __________

UMKC Department of Mathematics and Statistics

Math 210 CALCULUS I
Common Final Examination

Saturday, December 13, 2014

INSTRUCTIONS:

1. Write your name, the name of your instructor and the time your class meets on top.
2. Only scientific calculators may be used for this examination. Graphing calculators and electronic communication devices are NOT permitted in the room.
3. Make sure that your test contains all problems indicated below on this cover page.
4. On page 1 ONLY: Multiple choice problems; full credit will be given for a correct answer and no credit for an incorrect answer; no work needs to be shown.
5. On all other pages: **All work MUST be shown to receive full credit.**
6. Unless specifically asked not to simplify, all answers need to be simplified completely.
7. EXACT answers are expected, NO approximations (unless otherwise specified).
8. Write your final answer in the box provided.

Problem	Possible	Earned
1	20	
2	5	
3	9	
4	8	
5	6	
6	13	
7	9	
8	9	
9	16	
10	8	
11	12	
12	20	
13	10	
14	9	
15	10	
16	8	
17	9	
18	10	
19	9	
Total	200	

1. (20 pts) Determine the following limits. Choose "DNE" (Does Not Exist) as your answer only if your function has NO limit, finite or infinite. **Circle one of the indicated choices**. No work needs to be shown.

(a) (5 pts) $\lim_{x\to\infty} \dfrac{2x^3 - 10x + 2}{-2x^3 + x^2 - 1} =$

A. 1

B. -1

C. DNE

D. 2

E. -2

F. 0

(b) (5 pts) $\lim_{x\to\infty} \dfrac{3x^2 - x^3}{x^2 + 1} =$

A. -1

B. 1

C. DNE

D. ∞

E. $-\infty$

F. 3

(c) (5 pts) $\lim_{x\to 2} \dfrac{x^3 - 4x}{x^2 - 3x + 2} =$

A. 8

B. 4

C. 2

D. 1

E. ∞

F. DNE

(d) (5 pts) $\lim_{x\to 0^+} \dfrac{1}{e^x - 1} =$

A. 1

B. -1

C. DNE

D. ∞

E. $-\infty$

F. 0

2. (5 pts) Find a value of the constant k, if possible, that will make the function f continuous everywhere.

$$f(x) = \begin{cases} kx^2 & \text{for } x \leq 2 \\ 13x + k & \text{for } x > 2 \end{cases}$$

$k =$

3. (9 pts) Find the equation of the line tangent to the graph of $y = \dfrac{5}{x} + 3$ at $x = -1$.

$y =$

4. (8 pts) Use the limit DEFINITION of the derivative to show that the derivative of the function $f(x) = 2 - 5x^2$ is $f'(x) = -10x$.

5. (6 pts) Let $f(x)$ be a function with $f(2) = 5$ and $f'(2) = -1$. If $g(x) = x^3 f(x)$, find $g'(2)$.

$g'(2) =$

6. Compute the derivatives of the following functions; do not simplify the answer.

(a) (6 pts) $f(x) = \cos(3 \ln x)$

$f'(x) =$

(b) (7 pts) $g(x) = \dfrac{e^{3x}}{\tan(x)}$

$g'(x) =$

7. (9 pts) Let $f(x) = \sqrt{9 + 5x}$. Compute $f'(-1)$ and $f''(-1)$.

$f'(-1) =$

$f''(-1) =$

8. (9 pts) Find $\frac{dy}{dx}$ by differentiating implicitly: $x^3y^2 - 5x^2y + x = 1$.

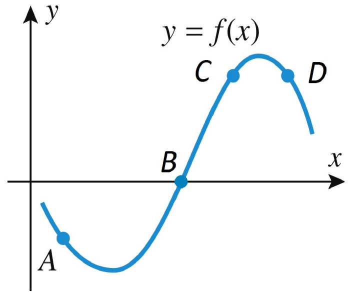
$\frac{dy}{dx} =$

9. (16 pts) Find the limits and show **all** the work involved.

(a) (8 pts) $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} =$

(b) (8 pts) $\lim_{x \to \infty} x \ln\left(1 + \frac{10}{x}\right) =$

10. (8 pts) Based on the graph sketched below, decide whether the following statements are true or false. Write either "TRUE" or "FALSE" next to each statement.



(a) $\dfrac{dy}{dx} > 0$ at the point D.

(b) $\dfrac{dy}{dx} = 0$ at the point B.

(c) $\dfrac{d^2y}{dx^2} > 0$ at the point A.

(d) $\dfrac{d^2y}{dx^2} > 0$ at the point C.

11. (12 pts) The function $s(t) = 9 - 9\sin(t)$ describes the position of a particle moving along a coordinate line, where s is in feet and t in seconds.

(a) (4 pts) Find the acceleration and velocity functions.

$v(t) =$

$a(t) =$

(b) (4 pts) At what times t with $0 \leq t \leq 2\pi$ is the particle stopped?

$t =$

(c) (4 pts) Is the particle **speeding up** or **slowing down** at the instant when $t = \dfrac{5\pi}{4}$? Write the answer in the box provided and provide a justification below.

12. A function $f(x)$ has $f'(x) = x^2(x-3)e^x$ and $f''(x) = x(x^2-6)e^x$ for $-\infty < x < \infty$.

(a) (8 pts) Find the open interval(s) on which f is increasing.

f is increasing on the interval(s):

(b) (4 pts) Find the x-coordinates of all points of relative minimum and all points of relative maximum of the function. Write "none" if no such points exist.

relative maximum at $x =$

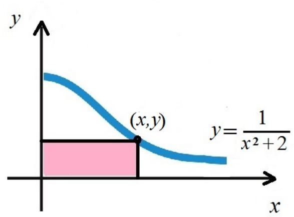
relative minimum at $x =$

(c) (8 pts) Find the open interval(s) where f is concave up.

f is concave up on the interval(s):

13. (10 pts) A rectangle is positioned as in the picture below such that one vertex is at the origin, the opposite vertex is on the graph of the function $y = \dfrac{1}{x^2+2}$, and the other two vertices are on the x-axis, respectively the y-axis.

Find the **maximum** possible area of such a rectangle.



maximum area=

14. (9 pts) A stone dropped into a pond sends out a circular ripple whose radius increases at a rate of 3 ft/s. How rapidly is the **area** enclosed by the ripple increasing when the radius is equal to 0.2 ft?

rate=

15. (10 pts) Evaluate the indefinite integral $\displaystyle\int \frac{3}{1+x^2} + \frac{\sec^2 x}{5} + \sqrt{2}\, dx$

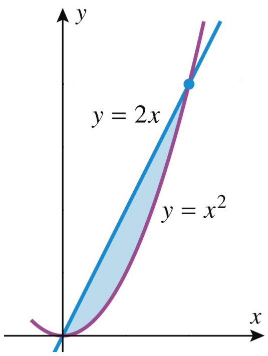
16. (8 pts) Solve the initial-value problem: $\displaystyle\frac{dy}{dx} = \sin\left(\frac{x}{2}\right) + 5, \quad y(0) = 1.$

$y(x) =$

17. (9 pts) Evaluate the definite integral $I = \displaystyle\int_0^{\ln 5} \frac{e^x}{e^x + 7}\, dx$ using a u-substitution as needed; show all work.

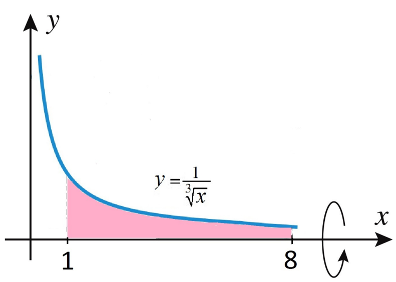
$I =$

18. (10 pts) Find the **area** of the region in the first quadrant enclosed between the graphs of $y = 2x$ and $y = x^2$.



AREA=

19. (9 pts) Consider the region between curve $y = \dfrac{1}{\sqrt[3]{x}}$ and the x-axis, over the interval $[1, 8]$. Find the **volume** of the solid obtained when this region is rotated about the x-axis.



VOLUME=