Name: ______________________________

Instructor: ______________________________

Time of your class: __________

UMKC Department of Mathematics and Statistics

Math 210 CALCULUS I
Common Final Examination

Saturday, December 7, 2013

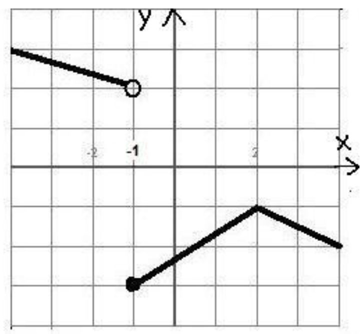
INSTRUCTIONS:

1. Write your name, the name of your instructor and the time your class meets on top.
2. Only scientific calculators may be used for this examination. Graphing calculators and electronic communication devices are NOT permitted in the room.
3. Make sure that your test contains all problems indicated below on this cover page.
4. On page 1 ONLY: Multiple choice problems; full credit will be given for a correct answer and no credit for an incorrect answer; no work needs to be shown.
5. On all other pages: **All work MUST be shown to receive full credit.**
6. Unless specifically asked not to simplify, all answers need to be simplified completely.
7. EXACT answers are expected, NO approximations (unless otherwise specified).
8. Write your final answer in the box provided.

Problem	Possible	Earned
1	20	
2	5	
3	5	
4	9	
5	14	
6	8	
7	5	
8	9	
9	16	
10	8	
11	8	
12	16	
13	8	
14	10	
15	8	
16	10	
17	9	
18	9	
19	5	
20	9	
21	9	
Total	200	

1. (20 pts) Determine the following limits. Choose "DNE" (Does Not Exist) as your answer only if your function has NO limit, finite or infinite. **Circle one of the indicated choices**. No work needs to be shown.

(a) (5 pts) Given $y = f(x)$ graphed below, $\lim_{x\to -1} f(x) =$



A. 1

B. -1

C. DNE

D. 2

E. -3

F. 0

(b) (5 pts) $\lim_{x\to\infty} \frac{x^3 - x + 2}{2 - x^4} =$

A. 1

B. -1

C. DNE

D. ∞

E. $-\infty$

F. 0

(c) (5 pts) $\lim_{x\to 2^+} \frac{x+3}{x-2} =$

A. 5

B. 1

C. DNE

D. ∞

E. $-\infty$

F. 0

(d) (5 pts) $\lim_{x\to 1} \frac{x^2 + x - 2}{x^2 - 1}$

A. DNE

B. ∞

C. 1

D. 1.5

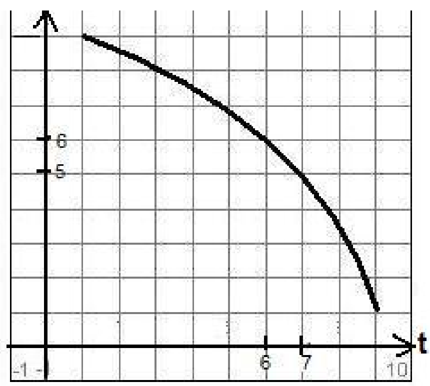
E. 1.6

F. 2

2. (5 pts) Is the following function continuous at $x = 5$? Explain briefly why or why not.

$$f(x) = \begin{cases} x^3 - 7 & \text{for } x \leq 5 \\ 15 - x & \text{for } x > 5 \end{cases}$$

3. (5 pts) The graph below shows the position versus time curve for a particle moving on a straight line. Estimate the instantaneous velocity at the time $t = 7$.



$v_{\text{inst}} \simeq$

4. (9 pts) Find the equation of the line tangent to the graph of $y = \dfrac{x+1}{x-1}$ at $x = 2$.

$y =$

5. (14 pts) Find the derivatives of the following functions:

(a) (4 pts) $f(x) = x^3 \sin(2x)$.

$f'(x) =$

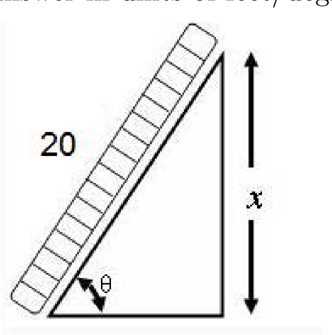
(b) (10 pts) $g(x) = 3\sin(\ln x) + e^{x^2} + 3$

$g'(x) =$

6. (8 pts) Let $f(x) = \sqrt[3]{x^2}$. Compute $f'(8)$ and $f''(8)$.

$f'(8) =$
$f''(8) =$

7. (5 pts) A 20 foot long ladder leans against a wall at an angle θ with the horizontal as shown in the figure. The top of the ladder is x feet above the ground. If the bottom of the ladder is pushed toward the wall, find the rate at which x changes with θ when $\theta = 60°$. Express the answer in units of feet/degree.



rate =

8. (9 pts) Find $\dfrac{dy}{dx}$ by differentiating implicitly: $x^6 + 3xy + y^2 = 17$.

$\dfrac{dy}{dx} =$

9. (16 pts) Find the limits and show **all** the work involved.

(a) (8 pts) $\lim_{x \to 0} \dfrac{e^x - 1}{\sin(\pi x)} =$

(b) (8 pts) $\lim_{x \to 0^+} x \ln x =$

10. (8 pts) Consider the function $f(x) = \sqrt{x}$. Show, using the **DEFINITION** of the derivative, that $f'(x) = \dfrac{1}{2\sqrt{x}}$.

11. (8 pts) The following information about a function $f(x)$ is known:

(a) $f(x)$ is defined and differentiable for all real x;

(b) $f(x) > 0$ for all x

(c) $f(x) = f(-x)$ for all x;

(d) $\lim_{x\to\infty} f(x) = 1$.

(e) $f'(0) = 0$, $f'(1) > 0$, $f'(2) = 0$ and $f'(3) < 0$.

Sketch below the graph of a function that *clearly* satisfies all these conditions.

12. (16 pts) Consider the function $f(x) = x^4 - 12x^2 + 12$.

(a) (8 pts) Find all open intervals where f is increasing.

f is increasing on the interval(s):

(b) (8 pts) Find all open intervals where f is concave up.

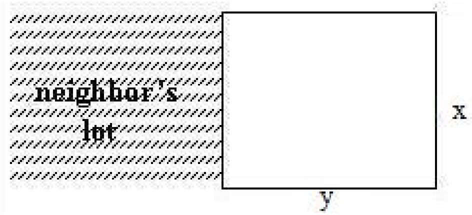
f is concave up on the interval(s):

13. (8 pts) Consider the function $f(x) = x + 2\cos(x)$, for $0 < x < \pi$. Find the points of relative extrema of $f(x)$.

relative maximum at $x =$

relative minimum at $x =$

14. (10 pts) A farmer wants to lay out a rectangular garden and enclose it with a fence. The garden is to have an area of 363 ft^2, and one of its sides is adjoining a neighbor's lot. The neighbor agrees to pay for half of the dividing fence. What should the dimensions of the garden be so that the farmer's total cost for the fence is minimized?



x=

y=

15. (8 pts) Gravitational force is inversely proportional to the square of the distance x between two objects, according to the formula:

$$F = \frac{81}{x^2} \qquad (N)$$

At what rate is the force decreasing at the instant when the distance between the objects is $x = 3$m, and the objects are moving away from each other at a rate of 0.3 m/s?

rate=

16. (10 pts) Evaluate the indefinite integral $\int \left(\frac{1}{3\sqrt{x}} + 5\sin x + e^{2x} \right) \, dx$

17. (9 pts) Evaluate the definite integral $I = \int_0^1 \frac{x}{(2x^2+1)^3} \, dx$ using a u-substitution; show all work.

$I =$

18. (9 pts) Find the area of the region enclosed between the graphs of $y = 1 - x^2$ and $y = 0$.

AREA=

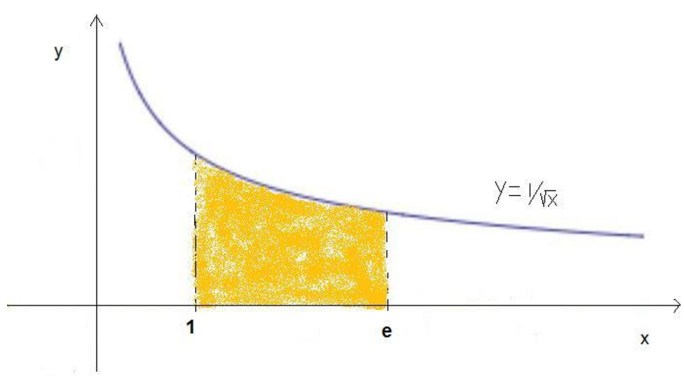
19. (5 pts) If $F(x) = \int_0^x \sin(t^2)dt$, use the Fundamental Theorem of Calculus (part II) to find $F'(x)$.

$F'(x) =$

20. (9 pts) A particle moves with a velocity $v(t)$ m/s along an s-axis, where $v(t) = t + \frac{1}{t^2+1}$. If $s(1) = \frac{1}{2}$, determine the position function $s(t)$.

$s(t) =$

21. (9 pts) Consider the region between curve $y = \frac{1}{\sqrt{x}}$ and the x-axis, over the interval $[1, e]$. Find the volume of the solid obtained when this region is rotated about the x-axis.



VOLUME=